Applied Mathematical Modelling 39 (2015) 1266–1274

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A low-rank tensor-based algorithm for face recognition

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ARTICLE INFO

Article history:
Received 30 May 2014
Received in revised form 24 July 2014
Accepted 12 August 2014
Available online 28 August 2014

Keywords:
Pattern recognition
Dimension reduction
Eigenvalue
Eigenvector
Singular values
Tensors

ABSTRACT

The face recognition problem arises in a wide range of real life applications. Our new developed face recognition algorithm, based on higher order singular value decomposition (HOSVD) makes use of only third order tensor. A convenient way of writing the commutativity of different modes of tensor-matrix multiplications leads to a new method that outperforms in terms of complexity another third order tensor method. The resulting algorithm is more successful (in terms of recognition rate) than the conventional eigenfaces algorithm. Its effectiveness is proved for two benchmark datasets (ExtYaleB and Essex datasets), which are ensembles of facial images that combine different modes, like facial geometries, illuminations, and expressions.

1. Introduction

In recent years, due to technical evolution, face recognition became an issue of increasing interest in a wide range of applications, such as security and surveillance problems, forensic problems, human–computer interface, access control problems, multimedia communications and so on. It is well known that human accuracy in identifying a face in a crowd is of 97.5%, being independent on expressions, head poses, illuminations conditions and other multiple factors. This robustness of the human perception is crucial to human social interaction. So far, this is an open problem in computer vision and pattern recognition.

Over the years, different types of algorithms have been developed for face recognition and, also, algorithms used in other branches of science were adapted to solve this problem [1,2].

Linear algebra, where the main objects are vectors and matrices, has provided with many valuable algorithms for this issue. From the Karhunen–Loeve transform [3,4] to principal component analysis [5,6], with the version of conventional eigenfaces, and its improved version, independent component analysis (ICA) [7], these algorithms take into account a single factor variations in image formation. When illumination, viewpoint, head pose, and expression vary, eigenfaces performs poorly.

Multilinear algebra, the algebra of higher-order tensors, offers powerful and sophisticated tools to approach a multifactor model of representation of images. A good survey on tensor is [8] where the authors provide an overview of higher order tensors, their decompositions and software packages for working with tensors [8,9].

We use in this paper a higher-order generalization of PCA (Tucker decomposition) and singular value decomposition (SVD) of matrices for computing principal components. If, for a matrix, the existence and uniqueness of SVD is assured, the situation for tensors are not the same: there is no true tensor SVD, with all good properties [10], there are many ways

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http://dx.doi.org/10.1016/j.apm.2014.08.003
0307-904X/

to decompose tensor orthogonally. We use in a convenient way this property in order to have representation that separates the different modes from the formation of facial images.

The rest of the paper is organized as follows. Section 2 reviews some related work; Section 3 summarizes the standard Eigenfaces algorithm whereas Section 4 recalls briefly the main results in tensor algebra that are relevant to our approach. Section 5 presents the multilinear analysis of the face recognition with the description of the proposed algorithm. Section 6 contains numerical experiments on the analysis of facial images from two benchmark datasets using our algorithm and the PCA. Section 7 concludes the paper.

2. Related work

Facial representation for recognition has been tackled by a family of PCA-based algorithm, such as Eigenfaces [6,5] and FisherFaces [11]. They compute the PCA by performing an SVD on $P \times N$ = data matrix of "vectorized" $N = m \times n$ pixel images of P people. This type of linear model is suited when the identity of the subject is the only variable that counts for image formation. In [12] it is proved that a natural representation of a collection of images is a third order tensor, rather than a simple matrix of vectorized images. N-mode analysis was first proposed by Tucker [13] and developed by Kapteyn et al. [14,15]. Also de Lathauwer in [15,16] develops the SVD for tensors, called higher order SVD (HOSVD).

Vasilescu and Terzopoulos proposed a "TensorFaces" representation in [17] which has several advantages over conventional Eigenfaces. They make use of 5-mode decomposition of a tensor (HOSVD for fifth order tensors), employing the Weizmann face database of 28 male subjects photographed in 15 different poses under 4 illuminations performing 3 different face expressions. Vasilescu in [18,19] addressing the motion analysis/synthesis problem, structured motion capture data in tensor form and developed an algorithm for extracting "human" motion signature. In [20], Vasilescu et al. introduced a tensor framework for image-based rendering and developed an algorithm called TensorTextures. The same authors introduced in [21] a multilinear generalization of ICA (MICA), and successfully applied this algorithm to a multimodel face recognition problem involving multiple people imaged under different viewpoints and illumination.

The 3-mode SVD facial representation technique (HOSVD for third order tensors) that we develop in this paper is for the case when the dataset is not fully organized. We mean that the information for a person is about only one attribute (or facial expression, or illumination, or head pose), and not about all of them as for the Weizmann face dataset. The obtained results are promising and comparable with the ones in MICA in [21] with n-order tensors.

3. Eigenfaces (PCA)

Although the Eigenfaces algorithm is already well-known, we will include a brief description of it for the sake of completeness. The eigenfaces are the eigenvectors of the covariance matrix of a set of faces [5,22]. These vectors are called eigenfaces because they resemble human faces when they are represented. These eigenfaces can be obtained by performing a mathematical process, namely the principal component analysis (PCA). The eigenvectors are chosen in descending order of their importance: the first component has the highest significance and so on. It must be taken into account that each principal component is orthogonal to all previous principal components. The idea of using principal components to represent human faces was developed by Sirovich and Kirby [4,6] and used by Turk and Pentland [5,22] for face detection and face recognition.

Every image, from the dataset, has the same resolution $M = n_1 \times n_2$ and is transformed into a vector $\Gamma_i \in \mathbb{R}^{M \times 1}$. Then we compute the average face vector Ψ of all N images and subtract the average face vector from all vectors $\varphi_i = \Gamma_i - \Psi$. The covariance matrix is $C = AA^T \in \mathbb{R}^{M \times M}$, with M the resolution of an image. Because in practice, M is very large, the computational effort to determine M eigenvalues and eigenvectors is huge. Thus, the idea is to reduce the amount of calculations by reducing the size. Let $L = A^TA, L \in \mathbb{R}^{N \times N}$. From all N vectors obtained, we keep only the first K vectors corresponding to the largest K eigenvalues. To identify a new image, Γ, we represent it using the eigenvectors $\{u_1, u_2, \ldots, u_K\}$. Thus, we have $\omega_i = u_i^T(\Gamma - \Psi), i = 1 : K$. These coefficients ω_i form the vector $\Omega^T = [\omega_1, \omega_2, \ldots, \omega_K]$. The vector Ω describes the contribution of each eigenface in representing the image Γ and is used to classify the new image Γ.

4. Fundamentals of tensor algebra

A N-order tensor is an object with N dimensions. If $N = 1$ (first order tensor) we have a vector and if $N = 2$ (second order tensor) we have a matrix. In the next section we will deal only with the case where $N = 3$. Hence, further, we consider a third order tensor $A \in \mathbb{R}^{l \times m \times n}$.

An element of tensor A is denoted by $A(i, j, k)$, where $1 \leqslant i \leqslant l,\ 1 \leqslant j \leqslant m$, and $1 \leqslant k \leqslant n$. We define the first mode fibers of a third order tensor A to be the column vectors $A(:, j, k)$, the second mode fibers as vectors $A(i, :, k)$, and the third mode fibers as vectors $A(i, j, :)$. Hence, fibers are characterized by fixing the index in all modes but one. Similarly, we define the slices of a tensor to be the matrix (for a third order tensor) obtained by fixing the index in one mode, namely we have the slices $A(i, :, :), A(:, j, :)$, and $A(:, :, k)$.