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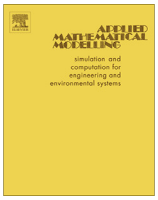

Multidimensional turbulence spectra – Statistical analysis of turbulent vortices ☆

Farideh Ghasempour *, Ronnie Andersson, Bengt Andersson

Department of Chemical and Biological Engineering, Chalmers University of Technology, SE-41296 Gothenburg, Sweden

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Strong nonlinear or very fast phenomena such as mixing, coalescence and breakup in chemical engineering processes, are not correctly described using average turbulence properties. Since these phenomena are modeled by the interaction of fluid particles with single or paired vortices, distribution of the properties of individual turbulent vortices should be studied and understood. In this paper, statistical analysis of turbulent vortices was performed using a novel vortex tracking algorithm. The vortices were identified using the normalized Q-criterion with extended volumes calculated using the Biot–Savart law in order to capture most of the coherent structure related to each vortex. This new and fast algorithm makes it possible to estimate the volume of all resolved vortices. Turbulence was modeled using large-eddy simulation with the dynamic Smagorinsky–Lilly subgrid scale model for different Reynolds numbers. Number density of turbulent vortices were quantified and compared with different models. It is concluded that the calculated number densities for vortices in the inertial subrange and also for the larger scales are in very good agreement with the models proposed by Batchelor and Martinez-Bazán. Moreover, the associated enstrophy within the same size of coherent structures is quantified and its distribution is compared to models for distribution of turbulent kinetic energy. The associated enstrophy within the same size of coherent structures has a wide distribution that is normal distributed in the logarithmic scale.

1. Introduction

A detailed description of turbulence spectra is needed to model and quantify many aspects of engineering flows e.g. the behavior of turbulent mixing, coalescence and break-up phenomena in chemical engineering processes [1]. One of the main mechanisms behind these phenomena is the interaction of fluid particles with single or paired turbulent vortices. The aforementioned phenomena usually occur very fast, often within a few milliseconds [2], and this time scale is equal to or smaller than the life time of turbulent vortices for many engineering applications [3]. Thus, the interaction cannot be modeled using the statistical mean properties of turbulence e.g. turbulent kinetic energy and dissipation rate [3–5]. Instead, the interaction might be better described by the distribution of the properties of single turbulent vortices, such as vortex size, lifetime, number density (the number of turbulent vortices per unit fluid volume), growth and dissipation rate, and the turbulent kinetic energy for vortices of different sizes at different locations.

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* Corresponding author. Tel.: +46 31 7722982.
E-mail address: farideh@chalmers.se (F. Ghasempour).

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Nomenclature

C	structure function parameter (–)
C_S	Smagorinsky constant (–)
E	turbulent energy spectrum ($m^3\ s^{-2}$)
L	largest turbulent vortices length scale and pipe length (m)
$\dot{n}$	number density (m^{-3})
Q	Q-criterion (s^{-2})
r	distance (m)
R	radius (m)
Re_λ	Taylor microscale Reynolds number
S	strain rate (s^{-1})
t	time (s)
u	fluctuating velocity ($m\ s^{-1}$)
$\bar{u}$	mean of fluctuating velocity ($m\ s^{-1}$)
x, y, z	coordinates
y^+	wall unit (–)

Greek letters

Δ	turbulence resolution filter size (m)
Ø	pipe diameter (m)
α	Kolmogorov constant (–)
ε	energy dissipation rate ($m^2\ s^{-3}$)
κ	wave number (m^{-1})
λ	vortex size (m)
λ_2	eigenvalue of velocity gradient tensor
ν	kinematic viscosity ($m^2\ s^{-1}$)
ρ	density ($kg\ m^{-3}$)
τ	vortex turnover time (s)
ω	vorticity (s^{-1})

Superscripts and subscripts

ind.	induced velocity in Biot–Savart integral
n	normalized
T	residual part in LES grid filter

Acronyms

LES	large eddy simulation
RANS	Reynolds-averaged Navier–Stokes
TKE	turbulent kinetic energy
SGS	subgrid scale
Ens	enstrophy

Moreover the detailed description of turbulence spectra helps to improve understanding of turbulence. The understanding of turbulence is a part of “wish list” suggested at Turbulence Colloquium in Marseille 2011 for current and future studies [6]. The intention of this research work is to improve the understanding of turbulence. For this purpose, a vortex-identification algorithm that allows the details of the single turbulent vortices to be visualized and their properties to be quantified, is required. Several vortex-identification algorithms are proposed in the literature. Many of the vortex identification algorithms identify the important regions include vortex cores, critical points for vector fields and regions that fit into predefined ideal shapes; but little work has been done to identify the real shape of three dimensional individual turbulent flow structures so far. In addition, there is still a need for developing novel methodologies that improve the study of three-dimensional turbulent structures’ properties.

Among the turbulent vortex properties, the vortex number density is critical for modeling of coalescence and break-up processes. Existing models predict different vortex number densities and they are only valid for the inertial subrange of the energy spectrum of turbulent flows. No model is available which is valid for a wider range of flow situations [7,8]. In addition the vortex number distributions should be used to derive expressions relating to the fractional rate of surface renewal and mass transfer coefficients across gas–liquid and solid–liquid interfaces [9]. For these reasons, a systematic evaluation of available models on vortex number density is required as a foundation for further investigations.

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