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Modelling bubble rise and interaction with a glass surface

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ABSTRACT

A theoretical model has been developed to analyse bubble rise in water and subsequent impact and bounce against a horizontal glass plate. The multiscale nature of the problem, where the bubble size is on the millimetre range and the film drainage process happens on the micrometre to nanometre scale requires the combined use of different modelling techniques. On the macro scale we solve the full Navier–Stokes equations in cylindrical coordinates to model bubble rise whereas modelling film drainage on the micro scale is based on lubrication theory because the film Reynolds number becomes much smaller than unity. Quantitative predictions of this model are compared with experimental data obtained using synchronised high-speed cameras. Video recording of bubble rise and bounce trajectories are combined with interferometry data to deduce the position and time-dependent thickness of the thin water film trapped between the deformed bubble and the glass plate. Bubble rise velocity indicated that the boundary condition at the bubble surface was tangentially immobile. Quantitative comparisons are presented for bubbles of different size to quantify similarities and differences.

1. Introduction

Interactions of soft materials such as drops and bubbles with solid surfaces occur in a wide variety of fields ranging from the manufacture of pharmaceuticals and detergents to water purification and mineral extraction. Modelling and numerical simulations of such systems present challenges because particle motions, interfacial interactions and deformations occur on different length scales. Identifying the bubble interface and its deformation and movement can be done using numerical techniques such as the volume of fluid (VOF) or level-set methods. When a bubble is very close to a solid surface or when two bubbles are very close to each other such techniques require very fine grids and the computational time can become unreasonably large. On the other hand, the use of lubrication theory to model the last stages of thin film drainage can be used if the local Reynolds number of the system is smaller than unity. However, such descriptions must be consistent with the large scale description of the centre of mass motion.

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Nomenclature

C_d	drag coefficient
d	diameter of the bubble (m)
F_{buoy}	buoyancy force (N)
F_{drag}	drag force (N)
g	gravitational acceleration (m/s^2)
H	initial separation (m)
H_{o}	typical vertical length scale (m)
$h_{\text{o}}(t)$	film thickness at the centre of interaction (m)
$h_m(t)$	minimum film thickness (m)
$h(r,t)$	film thickness (m)
n	refractive index of water
p	pressure (Pa)
r	radial coordinate (m)
$r_{\max}$	length of computational domain (m)
R	radius of the bubble (m)
R_{L}	Laplace radius (m)
Re	global Reynolds number
Re_{f}	film Reynolds number
t	time (s)
$\mathbf{u}$	velocity vector (m/s)
u	radial velocity (m/s)
v	vertical velocity (m/s)
$U(t)$	maximum velocity of the water in the film (m/s)
$V_{\text{CM}}(t)$	velocity of centre of mass of the bubble (m/s)
$V(r,t)$	velocity of the bubble surface (m/s)
V_{T}	approach velocity (m/s)
V_{o}	typical vertical velocity scale (m/s)
z	vertical coordinate (m)

Greek symbols

σ	interfacial tension (N/m)
ρ	water density (kg/m^3)
μ	water viscosity (Pa s)
λ	wavelength of the laser (m)
Π	disjoining pressure (Pa)

The behaviour of bubbles rising in water under buoyancy has been an active area of research. Terminal velocity data for bubbles of various sizes from different sources have been collated by Clift et al. [1]. Bubbles rising in ultra clean water attain larger velocities that correspond to a mobile (stress free) boundary condition at the bubble surface whereas the presence of contaminants renders the interface to be immobile, obeying the same no-slip boundary condition as that at a solid surface, and results in lower terminal velocities. Bubbles that are partially covered by surfactants or contaminants have intermediate terminal velocities. Exposure of the water to the atmosphere is enough for environmental impurities to contaminate the water over time and change the boundary condition at the bubble surface [2].

Studies on the effect of different surfactants at relatively low concentrations in water found a decrease in terminal velocity of a rising bubble as the surfactant concentration was increased until the velocity reached a constant value [3,4]. Works by Levich [5] and Cuenot et al. [6] indicate that at sufficiently low surfactant concentrations, bubble motion can cause a non-uniform surface distribution of surfactants, where the top or leading portion of a rising bubble remains clean while the surfactants are convected to the bottom, trailing part of the bubble surface. A simple model known as the “stagnant cap model” in which the surfactant-covered boundary is represented as an immobile boundary and surfactant-free surface is modelled as a mobile or stress free boundary can be used to calculate the bubble terminal velocity for different coverage ratios. This model has been solved analytically in the Stokes flow regime [7]. To test the boundary condition that should be applied at the bubble surface we employ ANSYS Fluent to compute terminal velocities under mobile and immobile boundary conditions assuming the bubble to remain spherical, as observed experimentally.

When a bubble collides with a surface, a multiscale problem arises as the separation between the bubble and the surface becomes much smaller than the bubble radius. Observing the small scale microhydrodynamics phenomena of film drainage between the bubble and the surface required different experimental techniques. Most experimental investigations of rising bubbles that collide with a horizontal planar surface were restricted to side view recordings that report deformations in the shape of the bubble and possible bounce trajectories [8–11], but corresponding information about the drainage of the thin

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