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Optimal task partition and state-dependent loading in heterogeneous two-element work sharing system

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ABSTRACT

Many real-world systems such as multi-channel data communication, multi-path flow transmission and multi-processor computing systems have work sharing attributes where system elements perform different portions of the same task simultaneously. Motivated by these applications, this paper models a heterogeneous work-sharing system with two non-repairable elements. When one element fails, the other element takes over the uncompleted task of the failed element upon finishing its own part; the load level of the remaining operating element can change at the time of the failure, which further affects its performance, failure behavior and operation cost. Considering these dynamics, mission success probability (MSP), expected mission completion time (EMCT) and expected cost of successful mission (ECSM) are first derived. Further, optimization problems are formulated and solved, which find optimal task partition and element load levels maximizing MSP, minimizing EMCT or minimizing ECSM. Effects of element reliability, performance, operation cost on the optimal solutions are also investigated through examples. Results of this work can facilitate a tradeoff analysis of different mission performance indices for heterogeneous work-sharing systems.

1. Introduction

This paper considers work-sharing systems with two processing elements jointly working on the same mission task. The two elements can be different in their parameters including performance, operation cost and failure behavior as they may be supplied from different vendors and/or have different exploitation history. Parameters of the same element may also change due to changing load levels during the mission. Particularly, when one of the two elements fails, the remaining element takes over the uncompleted task of the failed element upon finishing its own part. The load level of this remaining operating element can change at the time of the failure, which can further affect its performance, operation cost and failure behavior [1–3]. Depending on the task complexity, task partition and load levels of the two elements, mission performance indices including the probability, time and cost of mission task completion can vary. Thus, problems of finding optimal task partition and element load levels optimizing these mission performance indices for a specified task complexity are relevant and should be solved for reliable and cost-effective design of work-sharing systems.

The work sharing system considered in this paper is motivated by real-world systems such as multi-path flow transmission systems, multi-channel data communication systems, and parallel computing systems (multi-processor systems, computer grids or clusters), where elements work simultaneously on different portions of the same task to accomplish a specified mission [4].

Consider, for a specific example, a flow transmission system aimed at transferring a predetermined amount of material through two parallel channels (pipelines with pump stations). Each channel can work autonomously and contains specific equipment with given productivity, cost and reliability characteristics. The total instant throughput of the system does not matter, but the time needed to complete the entire transfer task as well as the task cost and success probability are important. The transfer task can be arbitrarily distributed between the channels and the load of pumps can be chosen from a set of available levels. In many real applications the change of load during the task performance is impossible or undesirable (possibility of human errors, overhead

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Acronyms and abbreviations: AFTM, accelerated failure-time model; cdf, cumulative distribution function; pdf, probability density function; MSP, mission success probability; ECSM, expected cost of successful mission; EMCT, expected mission completion time

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Nomenclature

W	amount of work in the mission task
R	mission success probability
E	expected mission completion time
C	expected cost of successful mission
x	fraction of mission task that should be performed by element 1
w_j	amount of work assigned to element j
$F_j(t)$, $f_j(t)$	*cdf*, *pdf* of time-to-failure of element j
L_j^-	load level of element j before failure of element $3-j$
L_j^+	load level of element j after failure of element $3-j$
$g_j(l)$	performance (productivity) of element j working with load level l
$\phi_j(l)$	cumulative time acceleration factor of element j working with load level l
T_j	time needed by element j to complete its part of the mission given the other element does not fail
$c_j(l)$	per unit time operation cost of element j working with load level l
$c_j(0)$	per unit time idle mode cost of element j
η_j, β_j	scale, shape parameters of baseline Weibull time-to-failure distribution for element j

and failures associated with transient processes, unavailability of personnel for constant process monitoring). If one of channels fails, the system inevitably requires personnel intervention. After the failure, the single available channel is assigned to transfer the remaining amount of the material and the load of this channel can be changed.

Note that the system considered is different from the traditional load sharing systems where the elements are loaded in a way that provides a desired level of cumulative system performance and the event of an element failure results in a higher load thus a higher failure rate to the remaining elements [5–7]. It is also different from the performance sharing systems in which the surplus instant performances of elements can be redistributed in a way that allows elements to meet individual demands [8–11]. The load levels of the elements in a work sharing system can be chosen without respect to cumulative system performance, but in a way that provides a desired balance among the mission success probability, expected cost and duration. The system considered also differs from an active redundant or hot standby system, in which multiple elements work on the same task in parallel but without any work sharing for the purpose of providing fast system recovery in the event of failures [12–14].

The considered work sharing system actually generalizes 1-out-of-2 warm standby system, where one element is online and working with the other element serving as a standby unit ready to take over the task in the event of the online element failure [15,16]. In other words, the 1-out-of-2 warm standby system is a special case of work sharing system considered when no task is initially assigned to one of the two elements.

Considerable research efforts have been expended in reliability modeling and optimization of traditional load sharing systems (e.g. [17–21]) and different types of standby systems (e.g. [22–27]). Refer to [7] and [28] for a review of these efforts. However little work is dedicated to modeling and optimizing reliability of work-sharing systems [4,29,30] and the existing work only focused on the task distribution problem. The possibility of uncompleted task reassignment as well as effects of state-dependent loading have not been addressed.

This paper makes original contributions by proposing a solution methodology to assess mission success probability (MSP), expected mission completion time (EMCT) and expected cost of successful mission (ECSM) of two-element heterogeneous work-sharing systems subject to uncompleted task reassignment and state-dependent element loading. Another contribution is to formulate and solve a set of optimization problems, which determine optimal task partition and element load levels with the objective to maximize MSP, minimize EMCT, or minimize ECSM subject to providing desired levels of MSP and EMCT. Examples are provided to demonstrate applications of the proposed evaluation and optimization methodology.

The remaining of the paper is organized as follows. Section 2 describes the two-element work sharing system model considered in this work. Section 3 presents evaluation of mission performance indices including MSP, EMCT and ECSM of the considered system. Section 4 illustrates the evaluation using examples. Section 5 presents formulation of related optimization problems. Effects of several element parameters on optimal solutions are also investigated through examples. Lastly, Section 6 gives conclusions and directions of future research.

2. Two-element system model

The system consists of two elements that have to perform jointly a specified amount of work W. The work is distributed between the elements based on a pre-specified partition when they are both functioning. The distribution or partition does not change if no element failures happen. In the case of failure of an element, the work uncompleted by this element is re-assigned to the remaining one.

When both elements are working, element j is subject to load level L_j^- (j=1, 2). Performance (or productivity) of element j working with load level l is $g_j(l)$**.** When the work is distributed such that the first element should perform $w_1=xW$ and the second element should perform $w_2=(1-x)W$ amount of work ($0\le x\le 1$), the time needed by element j to complete its part of the mission is

$$T_j = \frac{w_j}{g_j(L_j^-)} \tag{1}$$

and the mission time in the case of no failures is

$$T^* = \max\{T_1, T_2\}. \tag{2}$$

The mission succeeds either when any element j does not fail during time T_j or when element j fails before time T_j and element $3-j$ completes the mission. It may be decided to change the load level of element $3-j$ after the failure of element j from L_{3-j}^- to $L^{+}{}_{3-j}$.

Notice that when the work distribution parameter x=0 or x=1 the system considered reduces to 1-out-of-2 warm standby system, where one of the two elements initially performs no work and serves as a standby unit ready to take over the uncompleted task when the operating element fails.

3. Evaluating mission performance indices

In this section we derive mission success probability (MSP), expected mission completion time (EMCT), and expected cost of successful mission (ECSM) for the two-element system considered.

It has been shown through empirical studies that the load level