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Optimal order-replacement policy for a phase-type geometric process model with extreme shocks

Miaomiao Yu [a,b,*], Yinghui Tang [c], Wenqing Wu [c], Jie Zhou [a]

[a] School of Science, Sichuan University of Science and Engineering, 643000 Zigong, Sichuan, China
[b] Department of Electrical & Computer Engineering, University of Manitoba, R3T 5V6 Winnipeg, Manitoba, Canada
[c] School of Mathematics & Software Science, Sichuan Normal University, 610068 Chengdu, Sichuan, China

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ABSTRACT

A system is subject to random shocks that arrive according to a phase-type (PH) renewal process. As soon as an individual shock exceeds some given level the system will break down. The failed system can be repaired immediately. With the increasing number of repairs, the maximum shock level that the system can withstand will be decreasing, while the consecutive repair times after failure will become longer and longer. Undergoing a specified number of repairs, the existing system will be replaced by a new and identical one. The spare system for the replacement is available only by sending a purchase order to a supplier, and the duration of spare system procurement lead time also follows a PH distribution. Based on the number of system failures, a new order-replacement policy (also called (K,N) policy) is proposed in this paper. Using the closure property of the PH distribution, the long-run average cost rate for the system is given by the renewal reward theorem. Finally, through numerical calculation, it is determined an optimal order-replacement policy such that the long-run expected cost rate is minimum.

1. Introduction

Shock models have been widely studied in the past three decades and provided a realistic formulation for modeling certain reliability systems situated in random environment. In a broad sense, the term shock is used to describe the perturbations of the system caused by voltage fluctuations, temperature changes, human operational errors, etc. Under normal circumstances, the standard assumptions in shock models are that the system failure is related either to the cumulative effect of a number of shocks or that failure is produced by one large or extreme shock which surpasses a certain critical level. Recently, two major extensions to the traditional shock models are mixed and δ shock models. In the mixed shock model, the system breaks down when the cumulative shocks reach "some high" level or when a single "large" shock appears whichever comes first. While in the δ shock models, the system fails when the time between two consecutive shocks falls below a fixed threshold δ. Some classic and excellent research works about these topics can be found in Shanthikumar and Sumita [1,2], Gut and Hüsler [3], Mallor and Omey [4], Li and Kong [5], and references therein.

Like most repairable systems in the real world, the performance of a system that is subject to random shocks will deteriorate with age and usage, so that it will be more fragile and easier to break down after repairs. Thus the studies of optimal maintenance policy for various deteriorating systems have continuously attracted attention of many researchers. Using the

* Corresponding author at: School of Science, Sichuan University of Science and Engineering, 643000 Zigong, Sichuan, China. Tel.: +86 13540783127.
E-mail address: mmyu75@163.com (M. Yu).

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theory of geometric process (see [6,7]), the cumulative shock model for a degrading system was first investigated by Lam and Zhang [8]. We note that from the viewpoint of stochastic monotony, the geometric process repair model well described the degradation phenomenon arising in this system, and the analysis approach presented in this paper is simple and direct. Following groundbreaking work by Lam and Zhang, several researchers further studied some new maintenance model with random shocks (see, e.g. [9–14]), many fruitful theoretical results and interesting applications are reported in these research works.

On the other hand, a common feature in the existing literatures mentioned above is that most of these authors have invariably assumed that when a failed system needs to be replaced, there is an unlimited supply of spares readily available for replacements. It also means that system administrators store some spares in inventory so that they can be supplied immediately as soon as they are needed. Actually, in many situations such assumption might not be tenable (see, e.g. [15–19]). For example, when the spare is expensive and storage is costly, the cash flow will be tied up by spare parts inventory and lead to a substantial increase in system operating costs. Therefore, a more realistic model would be to consider a lead time for the supply of spares. Once we take account of the spare procurement lead time, determination of moment when maintenance should be performed will become the most important issue. Specifically, we should consider a strategy that determines when to order a spare and when to replace the failed system. In this paper, we revisit the extended extreme shock model, which was first introduced by Chen and Li [13], and reconsider the maintenance problem of degrading system under a new order-replacement policy, namely (K,N) policy. Here, the symbol N means that the system will be replaced at the time of the Nth failure, while K means that the order of the spare will be made at the end of the Kth repair $(1 \leqslant K < N)$. Thus, the major difference between our model and other models is that we first consider a random lead time for procurement of the spare system. In addition, we adopt PH distributions for random variables involved in the system. Hence, our model also provides much greater versatility due to the generality of the PH distributions. Further, employing some interesting closure properties of PH distribution, we also give the explicit expression for the long-run average cost function per unit time. Meanwhile, a numerical example is provided to illustrate the computational procedure for determining the optimal order-replacement policy.

The rest of this paper is organized as follows. In Section 2 the assumptions of the model are stated. The mean time between two consecutive replacements is discussed in Section 3. Section 4 is dedicated to derive the explicit expression of the long-run average cost rate function under maintenance policy (K,N). Concluding remarks are offered in Section 5.

Given that the continuous PH distribution and the concept of geometric process play a fundamental role in our study, we give their formal definitions as follows.

Definition 1 (*Neuts [20]*). A distribution $F(\cdot)$ on $[0,\infty)$ is of phase-type with representation $(\boldsymbol{\alpha},\boldsymbol{T})$, if it is the distribution of the time until absorption in a Markov process on the states $\{1,\ldots,m,m+1\}$ with infinitesimal generator

$$\begin{pmatrix} \boldsymbol{T} & \boldsymbol{T}^0 \\ \boldsymbol{0} & 0 \end{pmatrix}$$

and initial probability vector $(\boldsymbol{\alpha},\alpha_{m+1})$, where $\boldsymbol{\alpha}$ is a row vector of order m. Assuming that the states $\{1,\ldots,m\}$ are all transient, and the state $m+1$ is absorbing. Hence the matrix $\boldsymbol{T}$ can be interpreted as the rate transition matrix among the transient states, while $\boldsymbol{T}^0$ represents the column vector of absorption rates. The matrix $\boldsymbol{T}$ of order m is non-singular with negative diagonal entries and non-negative off-diagonal entries and satisfies $-\boldsymbol{Te}=\boldsymbol{T}^0$. The distribution $F(\cdot)$ is given by $F(x)=1-\boldsymbol{\alpha}\exp(\boldsymbol{T}x)\boldsymbol{e},\ x \geqslant 0$, where $\boldsymbol{e}$ denotes a column vector of ones of appropriate dimension.

Remark 1. We note that the distribution function $F(x)$ has a jump of height α_{m+1} at $x=0$. In order to ensure the reasonableness of our model assumption, throughout this paper we shall assume that $\alpha_{m+1}=0$ so that $F(x)$ does not have an atom at 0.

Definition 2 (*Ross [21]*). . Given two random variables ξ and $\tilde{\xi}$, ξ is said to be stochastically larger than $\tilde{\xi}$ or $\tilde{\xi}$ is stochastically smaller than ξ, if $P\{\xi > c\} \geqslant P\{\tilde{\xi} > c\}$, for all real c. It is denoted by $\xi \geq_{st} \tilde{\xi}$ or $\tilde{\xi} \leq_{st} \xi$. Furthermore, we say that a stochastic process $\{X_n, n=1,2,\ldots\}$ is stochastically decreasing (increasing) if $X_n \geq_{st} (\leq_{st}) X_{n+1}$ for all $n=1,2,\ldots$.

Definition 3. A stochastic process $\{\xi_n,\ n=1,2,\ldots\}$ is a geometric process, if there exists a real $a>0$ such that $\{a^{n-1}\xi_n,\ n=1,2,\ldots\}$ forms a renewal process. The real a is called the ratio of the geometrical process (see, e.g. [6,7,22] for more details).

Obviously, from Definition 3, we have:

If $a>1$, then the geometric process $\{\xi_n,\ n=1,2,\ldots\}$ is stochastically decreasing.

If $0<a<1$, then the geometric process $\{\xi_n,\ n=1,2,\ldots\}$ is stochastically increasing.

If $a=1$, then the geometric process $\{\xi_n,\ n=1,2,\ldots\}$ is a renewal process.

For further use in sequel, we introduce the following notations. Let $\boldsymbol{e}_m$, $\boldsymbol{I}_m$ and $\boldsymbol{0}_{m\times n}$ denote, respectively, the column vector of dimension m consisting of 1's, an identity matrix of dimension m and a zero matrix of dimension $m \times n$. $\otimes$ and $\oplus$ are symbols of the Kronecker product and sum of matrices.