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# A new RBDO method using adaptive response surface and first-order score function for crashworthiness design

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ABSTRACT

This study presents a new Reliability-based Design Optimization method using adaptive response surface and first-order score function analysis for complex system design optimization considering the variability of design variables. The adaptive response surface using Bayesian metric and Gaussian process based model bias correction method, is developed to represent the true performance functions and replace the true limit state function. First-order score function analysis is exploited to compute the sensitivities of probabilistic responses with respect to the design variables, which are the mean values of the random variables. Numerical results indicate that the proposed methods can produce the best response surface and estimate the sensitivities of probabilistic responses accurately. The proposed methodology is demonstrated by a vehicle crashworthiness design optimization problem with full frontal and offset frontal impacts.



## 1. Introduction

Reliability-based design optimization (RBDO) has gained more attention recently. It has been widely applied to various engineering design problems to consider the variabilities of design variables [1–6]. The commonly used RBDO methods can be classified into two categories: One is the most probable point (MPP)-based methods such as first-order reliability method (FORM), second-order reliability method (SORM), reliability index approach (RIA) and performance measure approach (PMA) etc., where the accurate sensitivities of performance function are required in complex engineering systems [7–11]. In the literature, MPP-based reliability design optimization is commonly used but it has some challenges, e.g., it may be unable to find an MPP. It also has difficulty to find and deal with problems which have multiple MPPs. The other is sampling-based methods which estimate the probability of failure and the sensitivities of performance function with selected samples by using sampling methods such as Monte Carlo simulation (MCS). For the former, the sensitivities of performance function are often required among the MPP-based methods [7]. For the latter, the sampling-based RBDO method by using MCS is general for obtaining the reliability of a complex system. However, in most practical engineering systems, the sensitivities of performance function are either unavailable or extremely difficult to obtain, since it is computation intensive for large-scale systems, especially for vehicle crashworthiness design. To reduce the computational expense for obtaining accurate response of performance function, the response surface-based RBDO becomes a common tool in practical engineering applications.

One of the major deficiencies of using the response surface in design optimization is its poor accuracy it may often encounter. In essence, the response surface is a simplified representation of a complex physical phenomenon. It ignores the data uncertainty, which may result from high non-linearity of the response of impact problem, imperfection of the numerical simulation and uncertainties in simulation [12–14]. As a result, the selection of response surface based on conventional metric calculated at sampling points, e.g., mean square error (MSE) etc., may not provide sufficient information on the evaluation of the accuracy of a response surface for the use in design optimization problems [15]. In this context, Yang et al. [16] and Gu et al. [17] recommended the use of second order polynomial regression and moving least square regression, respectively. Kurtaran et al. [18] suggested the successive response surface approximation, and Fang et al. [19] recommended the use of radial basis function. Although each response surface has its advantages, there is no common agreement on which model is the best. Shi et al. [20] proposed a Bayesian metric to help selecting the best available response surface in a model library considering data uncertainty. It is used as a measure to estimate the quality of a response surface in the presence of data uncertainty. The response surface with larger value of Bayesian metric implies that it is the most probable model in view of

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**Nomenclature**

| | |
|---|---|
| RBDO | Reliability-based Design Optimization |
| GP | Gaussian Process |
| SSR | Subset Selection Regression |
| RBF | Radial Basis Function |
| RBF-GP | Radial Basis Function with Gaussian basic function |
| RBF-MQ | Radial Basis Function with Multiquadric basic function |
| DOE | Design of Experiment |
| RMSE | Root Mean Square Error |
| N | Number of design variable |
| **I** | Any prior information |
| $prob(\bullet)$ | Probability measure |
| $m$ | Number of model parameters |
| $N$ | Sample size |
| $\mathbf{a}$ | Parameter of response surface **A** |
| $\hat{\mathbf{a}}$ | Parameter space of response surface **A** |
| $\sigma$ | Data uncertainty |
| $\varepsilon$ | Random gaussian noise |
| $\mathbf{K}$ | Matrix of Bayesian metric |
| $K_{i,j}$ | Element of matrix **K** |
| In $Q$ | Bayesian metric |
| $y^m(x)$ | Computer model (or response surface) |
| $y^e(x)$ | Physical observations (or high-fidelity model) |
| $\delta(x)$ | Bias correction function |
| $\varepsilon$ | Experimental error of the physical observation |
| $\Delta(x)$ | Updated bias correction function |
| $y(\mathbf{x})$ | Actual response |
| $\mathbf{h}^T(\mathbf{x})$ | Vector of the mean function of a GP model |
| $\boldsymbol{\beta}$ | Regression coefficients for the mean function of a GP |
| $\alpha^2$ | Constant variance of the GP model |
| $R(\mathbf{x}, \mathbf{x}')$ | Correlation function of the responses at points **x** and **x**′ |
| $\boldsymbol{\omega}$ | Vector of roughness parameters |
| MLE | Maximum likelihood estimates |

the same data and all known prior information. This metric proved to be a useful means to select the most likely response surface for engineering application with prior data uncertainty. However, the question of how to determine the prior information, i.e., data uncertainty, is not addressed. In addition, although Bayesian metric is effective to select the most probable response surface among candidates, the accuracy of the response surface may still depend on the availability of the response surfaces.

To address the data uncertainty and the accuracy problems, a Gaussian Process (GP) based model bias correction method is employed to quantify data uncertainty and also to improve its predictive capability. The GP for model bias correction is first used to address this issue of quantification of data uncertainty, and the Bayesian metric is utilized to select the most probable response surface. After that, the accuracy of the response surface is further improved by using the bias correction function. One interpretation of the model bias correction approach is that it captures the potential model error due to the use of incorrect modeling method, which often cannot be compensated by other means. It attempts to correct for any inadequacy of the model which is revealed by a discrepancy between the observed data and the model predictions from even the best-fitting parameter values [21].

Furthermore, in sampling-based RBDO method respect, MCS can be easily incorporated with response surface for calculating the sensitivities of reliability of complex systems for RBDO problems. However, even if an accurate response surface is available, the sensitivities of reliability obtained from the finite difference method (FDM) may be inaccurate as a noisy response is considered [22]. Using MCS method to calculate the sensitivities of probabilistic responses obtained by FDM is very time consuming if not impossible.

In this paper, the GP for model bias correction is first used to address this issue of quantification of data uncertainty, and the Bayesian metric is utilized to select the most probable response surface. The accuracy of the response surface is further improved by using the bias correction function. In addition, first-order score function is exploited to calculate the sensitivities of probabilistic response with respect to the design variables, which are the mean values of the random variables, to solve RBDO problem. The major advantage is that the sensitivity is computed analytically by taking derivatives of the probability density function, there is no approximation to calculate the sensitivities of the probabilistic responses.

The remainder of this paper is organized as follows: Section 2.1 briefly reviews the theory of RBDO. Section 2.2 proposes an adaptive response surface method by Bayesian metric and model bias correction and illustrates it with an analytical example, and followed by first-order score function for calculating sensitivities of probabilistic response in Section 2.3. A vehicle example with full-frontal and offset-frontal impacts is used to demonstrate the proposed methodology in Section 3. The summary is given in the end.

## 2. A new RBDO methodology

### 2.1. RBDO formulation

A typical RBDO problem, which involves probabilistic constraints, is generally formulated as:

$$\begin{aligned}&\text{minimize } F(\mathbf{d}, \mathbf{X})\\&\text{subject to } P\left[G_j(\mathbf{d}, \mathbf{X}) < 0\right] \le P_{F_j}^{tar}, j = 1, \ldots, nc\\&\qquad \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \mathbf{d} \in \mathbf{R}^{ndv}, \textit{ and } \mathbf{X} \in \mathbf{R}^n \end{aligned} \quad (1)$$

where $F$ and $G_j$ are the objective and constraint functions, respectively, $\mathbf{d}$ is the vector of design variables, $\mathbf{X}$ is the vector of design variables, $P_{F_j}^{tar}$ is the target probability of failure for $j^{th}$ constraint, and $nc$, $ndv$, and $n$ are the number of probabilistic constraints, design variables, and random variables, respectively. Note that the design variables $\mathbf{X}$ involves deterministic design variables and random design variables. To be more straightforward, $\mathbf{X}$ is only denoted as random design variables in this paper. The reliability is defined as 1-$P_F$, where $P_F$ is the probability of failure obtained by taking the expectation of an indicator function [23]:

$$P_F(\boldsymbol{\theta}) \equiv P_{\boldsymbol{\theta}}[\mathbf{X} \in \Omega_F] = \int_{\mathbf{R}^n} I_{\Omega_F}(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}; \boldsymbol{\theta}) d\mathbf{x} = E_{\boldsymbol{\theta}}[I_{\Omega_F}(\mathbf{X})] \quad (2)$$

where $\Omega_F \equiv \{\mathbf{x}: G(\mathbf{x}) < 0\}$ is the failure set for the component reliability analysis, $\boldsymbol{\theta}$ is a vector of distribution parameters, $f_{\mathbf{X}}(\mathbf{x}; \boldsymbol{\theta})$ is a joint probability density function of $\mathbf{X}$, $E_{\boldsymbol{\theta}}$ is the expectation operator, and $I_{\Omega_F}(\mathbf{x})$ is an indicator function expressed as:

$$I_{\Omega_F}(\mathbf{x}) = \begin{cases} 1, & \mathbf{x} \in \Omega_F \\ 0, & \mathbf{x} \in \Omega/\Omega_F \end{cases}; \qquad \mathbf{x} \in \mathbf{R}^n \quad (3)$$

where $\Omega$ is the design space.

The RBDO problem in (1) has been addressed extensively in the literature. Among those, Most Probable Point (MPP) has gained most attention in recent years [2–4]. In this research, instead,