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Structure preserving eigenvalue embedding for undamped gyroscopic systems

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ABSTRACT

This paper concerns the eigenvalue embedding problem of undamped gyroscopic systems. Based on a low-rank correction form, the approach moves the unwanted eigenvalues to desired values and the remaining large number eigenvalues and eigenvectors of the original system do not change. In addition, the symmetric structure of mass and stiffness matrices and the skew-symmetric structure of gyroscopic matrix are all preserved. By utilizing the freedom of the eigenvectors, an expression of parameterized solutions to the eigenvalue embedding problem is derived. Finally, a minimum modification algorithm is proposed to solve the eigenvalue embedding problem. Numerical examples are given to show the application of the proposed method.

1. Introduction

Obtaining highly accurate analytical structural models is necessary for analyzing, predicting and controlling the dynamic performance of complex structures during analysis and design. Finite element method is an efficient model building tool widely used by engineers, designers and analysts. By finite element method, the vibrating phenomenon of the undamped gyroscopic systems, such as rotors of the generator, solar panels on the satellite and so on, may be modelled by the following second-order ordinary differential system

$$M_a\ddot{q}(t) + G_a\dot{q}(t) + K_a q(t) = 0, \tag{1}$$

where $q(t) \in \mathbb{R}^n$ is a displacement vector depending on time t, n stands for the number of degrees of freedom of the analytical model, and $M_a, G_a, K_a \in \mathbb{R}^{n\times n}$ are the analytical mass, gyroscopic and stiffness matrices, respectively. In general, M_a is symmetric and positive definite, denoted by $M_a > 0$, K_a is symmetric and positive semidefinite, denoted by $K_a \geqslant 0$, and G_a is skew-symmetric, i.e., $G_a^T = -G_a$. (1) is usually known as the finite element model or the analytical model of the undamped gyroscopic systems. For simplicity, we will denote the model by the matrix triple (M_a, G_a, K_a). If $M_a > 0, K_a \geqslant 0$, and $G_a = D = D^T$, then (1) is usually called the analytical model of the viscous damping systems. In this paper, we will assume throughout that $M_a > 0$, $K_a > 0$ and $G_a^T = -G_a$. It is well known that if $q(t) = xe^{\lambda t}$ is a fundamental solution of (1), then the scalar λ and the vector x must solve the following quadratic eigenvalue problem

$$Q_a(\lambda)x = 0, \tag{2}$$

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where

$$Q_a(\lambda) = \lambda^2 M_a + \lambda G_a + K_a. \tag{3}$$

Lancaster [1] and Gohberg et al. [2,3] developed the spectral theory for the viscous damping systems. Tisseur and Meerbergen [4] surveyed many applications, mathematical properties, and a variety of numerical methods for the quadratic eigenvalue problem. Recently, Chu and Xu [5] characterized a real-valued spectral decomposition for the viscous damping systems and described its applications to the quadratic inverse eigenvalue problems. The mathematical properties and numerical methods for the quadratic eigenvalue problem of gyroscopic systems can be found in some references. See, for example, [6–10] and the references contained therein. Based on a real standard pair and a skew-symmetric parameter matrix, Jia and Wei [11], and Mao [12] developed a real-valued spectral decomposition for the undamped gyroscopic systems and discussed the applications of the spectral decomposition to the inverse eigenvalue problems of the undamped gyroscopic systems, respectively.

Since K_a(stiffness matrix) and M_a(mass matrix) are positive definite, it is well known that the $2n$ eigenvalues of $Q_a(\lambda)$ are purely imaginary and semisimple, denoted by $\{\pm i\omega_j\}_{j=1}^n$ with $\omega_j > 0$. The dynamical behavior of the gyroscopic systems is determined by their natural frequencies and corresponding mode shapes, that is, the eigenvalues and eigenvectors of the pencil $Q_a(\lambda)$. Owing to the complexity of the structure, the finite element model is an approximate discrete analytical model of the continuous structure. Natural frequencies and mode shapes of the analytical model do not match very well with experimentally measured frequencies and mode shapes obtained from a real-life vibrating test. Thus, updating the existing dynamic model on the basis of modal test data is very important for predicting actual behavior of the structure precisely via the structural dynamic model. The updated model may be considered a better representation of the actual structure than the original finite element model, and can be used with more confidence to analyze, predict and control the dynamic responses of the structure.

Over the past years, finite element model updating problem has received considerable discussions. Various methods have been developed for correcting analytical models to predict test results more closely. Interested readers are referred to survey papers [13,14]. A detailed theoretical analysis of model updating techniques can be found in the seminal book [15]. The existing direct methods, see, e.g., [16–21], can reproduce the given set of measured data, but cannot guarantee that the remaining eigenvalues and eigenvectors of the analytical model remain unchanged. Furthermore, these methods deal mainly with the inverse eigenvalue problem in the liner pencil $\lambda M - K$ rather than quadratic inverse eigenvalue problem for the pencil (3). Based on the spectral theory of matrix polynomials and the structure-preserving similarity, Lancaster et al. [22–24] considered the inverse spectral problems for damped vibrating systems. With only partially prescribed eigenpairs available, Chu et al. [25], Kuo et al. [26], and Cai et al. [27] discussed the quadratic inverse eigenvalue problems for viscous damping vibration systems by using matrix decompositions. In order to maintain symmetry and positive definiteness in the coefficient matrices, the generalized Newton approach [28], the alternating direction method of multipliers [29] and the interior-point approach [30] have been presented for solving the quadratic inverse eigenvalue problems of the viscous damping systems. However, these methods can not guarantee that extra, spurious modes are not introduced into the range of the frequency range of interest. The challenge, known as the no spill-over phenomenon in the engineering literature, is that in updating an existing analytical model it is often desirable that the current vibration parameters not related to the newly measured parameters should remain invariant. Applying feedback control, partial pole assignment techniques have been developed for the viscous damping systems [31–34] and the undamped gyroscopic systems [35], respectively. However, these approaches destroy the symmetric or skew-symmetric structure of the coefficient matrices in the original analytical models. Recently, Carvalho et al. [36] proposed a direct method for undamped model updating with no spill-over. Chu et al. [37–39] considered the quadratic model updating with no spill-over for the viscous damping systems. However, It may be found in practical computation that only a small number of eigenvalues obtained by the analytical models are "troublesome". Thus, it is desirable in engineering applications that these troublesome eigenvalues are reassigned to suitable locations chosen by the designer, while keeping the remaining large number of eigenvalues unchanged and making minimal changes in the analytical model. Such a problem in structural dynamic model updating is known as the eigenvalue embedding problem (EEP).

Let $\pm i\omega_1, \pm i\omega_2, \ldots, \pm i\omega_n (\omega_j > 0, j = 1, 2, \ldots, n)$ and $x_{1R} \pm ix_{1I}, x_{2R} \pm ix_{2I}, \ldots, x_{nR} \pm ix_{nI}$ be the $2n$ eigenvalues and corresponding eigenvectors of the analytical model (M_a, G_a, K_a). In this paper, we consider the following eigenvalue embedding problem of the undamped gyroscopic systems.

Problem EEP. Given an analytical model (M_a, G_a, K_a) with M_a, K_a symmetric positive definite and G_a skew-symmetric, the self-conjugate eigenvalue subset $\{\pm i\omega_1, \pm i\omega_2, \ldots, \pm i\omega_p\}$ and the corresponding eigenvector set $\{x_{1R} \pm ix_{1I}, x_{2R} \pm ix_{2I}, \ldots, x_{pR} \pm ix_{pI}\}$, and the measured natural frequency set $\{\mu_1, \mu_2, \ldots, \mu_p\}$, update the analytical model (M_a, G_a, K_a) to (M, G, K) of the same structure such that

(1) $M = M^T,\ G = -G^T,\ K = K^T$.
(2) The subset $\{\pm i\omega_1, \pm i\omega_2, \ldots, \pm i\omega_p\}$ is replaced by $\{\pm i\mu_1, \pm i\mu_2, \ldots, \pm i\mu_p\}$ as $2p$ eigenvalues of the updated model (M, G, K).
(3) The remaining (unknown) $2(n-p)$ eigenpairs of the updated model (M, G, K) are the same as those of the original model (M_a, G_a, K_a).

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