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Study on optimal kinematic synthesis of cam profiles for engine valve trains

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ABSTRACT

Kinematic cam profile synthesis for engine valve trains to satisfy the requirements of a large lift area and small acceleration magnitudes is a multi-objective optimization problem. This paper proposed that its Pareto optimal solution set is constructed with constant acceleration profiles. For the definite design of an engine valve train, its Pareto front is a function of the time durations of the positive and negative accelerations. The relationships of positive acceleration, negative acceleration, and lift area for a constant acceleration cam profile are analyzed, and the results are applied to discuss the characteristics of polynomial cam profiles.

1. Introduction

When designing a cam profile for an internal combustion engine, it is desirable to make the follower lift area as large as possible so as to improve the gas exchange capacity of the valves. In the meantime, it is also desirable to reduce the positive and negative accelerations in order to improve the dynamic properties of valve trains, because large accelerations will result in high loads on parts, residual vibrations in the system, and potentially the separation of components [1–4]. Because these factors conflict with one another, coordination is necessary in optimization. Thus, cam profile synthesis to satisfy the requirements of a large lift area and small acceleration magnitudes is a multi-objective optimization problem, and it is important to seek the Pareto solution set for valve train design.

Cam profile optimization has been studied extensively over the past decades, taking advantages of the tremendous advances in mathematical tools, especially the splines [5–9]. For the cam follower systems used in automotive engine valve trains, cam profiles must be precisely refined to ensure the desired gas exchange and dynamic properties. Some research has focused on the optimum design for these special requirements. For instance, Jeon proposed a two-step optimization technique to design an optimal cam profile. In the first step, an attempt was made to maximize valve lift area satisfying certain design constraints. In the second step, minor modifications of the cam developed in the first step were made in order to reduce the cam acceleration while maintaining the maximized valve lift area [2]. Li et al. found the optimal relationships among valve cam profile, cam speed, and stiffness based on a vast amount of calculations for a valve train model of single-mass vibration and a dynamic model of finite elements [10]. Lu et al. discussed the objectives and constraints of cam profile optimization and pointed out some problems of valve train design [11]. Ernst et al. presented methods based on parameter optimization and the minimum principle by Pontrjagin, and applied them to cam lobe and valve spring

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optimization aimed at reducing oscillation amplitudes [12]. Seldom does the literature discuss the Pareto solution of the multi-objective optimization problem of cam profile design for engine valve trains.

This paper proposes that cam profiles whose follower accelerations are constant in the positive and negative directions respectively (called constant acceleration profiles) construct the kinematic Pareto optimal solution set with regard to the objectives of maximizing the lift area of the follower and minimizing the magnitudes of the follower's positive and negative accelerations. The relationships of the lift area and the acceleration values are discussed, and the results are applied to discuss the characteristics of polynomial cam profiles.

2. Description of a non-dimensional cam curve

Cam profiles in a non-dimensional form of the follower motion are usually used in practice in the design of a cam mechanism. Such a cam profile can be defined by Eqs. (1) and (2):

$$\begin{aligned} &T = \varphi/\varphi_0, \\ &S = S(T) = Y(\varphi)/h \quad (0 \leqslant T \leqslant 1,\ 0 \leqslant S \leqslant 1,\ S(0) = 0,\ S(1) = 1), \end{aligned} \tag{1}$$

where S and T are the non-dimensional displacement and time, φ and Y are the rotation angle of the cam and the follower lift corresponding to this angle, and h and φ_0 are the maximal follower lift and the cam angle corresponding to the maximal follower lift, respectively.Differentiating S with respect to T, the velocity, acceleration, and jerk in non-dimensional form, denoted as V, A, and J, can be obtained as follows:

$$\begin{aligned} &V = V(T) = dS/dT, \\ &A = A(T) = d^2S/dT^2, \\ &J = J(T) = d^3S/dT^3. \end{aligned} \tag{2}$$

The lift area, that is, the area enclosed by the curve of the non-dimensional displacement and the axis of non-dimensional time can be denoted with the non-dimensional displacement as follows:

$$SS = \int_0^1 S(T)dT. \tag{3}$$

The relations between the real motion of the cam profile and the non-dimensional form are shown in Eq. (4), as follows:

$$Y = hS(T), \quad \frac{dY}{d\varphi} = \frac{h}{\varphi_0}V(T), \quad \frac{d^2Y}{d\varphi^2} = \frac{h}{\varphi_0^2}A(T), \quad \frac{d^3Y}{d\varphi^3} = \frac{h}{\varphi_0^3}J(T). \tag{4}$$

In order to describe discontinuous acceleration segments, the operator $\langle\ \rangle$ is induced and defined as follows [13]:

$$\begin{cases} \langle x \rangle = 0, & x < 0; \\ \langle x \rangle = x, & x \geqslant 0. \end{cases} \tag{5}$$

If a curve segment can be described as

$$\langle T - T_i \rangle^n = \begin{cases} 0, & T < T_i; \\ (T - T_i)^n, & (T \geqslant T_i, n > 0). \end{cases} \tag{6}$$

Its derivatives and integral can be calculated as follows:

$$\frac{d\langle T - T_i \rangle^n}{dT} = n\langle T - T_i \rangle^{n-1}, \tag{7}$$

$$\int \langle T - T_i \rangle^n dT = \frac{\langle T - T_i \rangle^{n+1}}{n+1} + C, \tag{8}$$

where C is a integral constant.

3. Pareto optimal set of kinematic cam profile synthesis for valve trains

3.1. Multi-objective optimization problem

In general, a multi-objective minimization problem with m design variables and n objectives is modeled as follows [14]:

$$\begin{aligned} \text{Minimize} \quad & \boldsymbol{y} = f(\boldsymbol{x}) = \{f_1(\boldsymbol{x})\, f_2(\boldsymbol{x}), \ldots, f_n(\boldsymbol{x})\}; \\ \text{where} \quad & \boldsymbol{x} = (x_1, x_2, \ldots, x_m) \in \boldsymbol{X}, \\ & \boldsymbol{y} = (y_1, y_2, \ldots, y_n) \in \boldsymbol{Y}, \end{aligned} \tag{9}$$

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