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Generation and suppression of a new hyperchaotic nonlinear model with complex variables

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ABSTRACT

In this paper, we introduce a new hyperchaotic complex Chen model. This hyperchaotic complex system is constructed by adding a complex nonlinear term to the third equation of the chaotic complex Chen system with consideration it's all variables are complex. The new system is a 6-dimensional continuous real autonomous hyperchaotic system. The properties of this system including invariance, dissipation, equilibria and their stability, Lyapunov exponents, Lyapunov dimension, bifurcation diagrams and hyperchaotic behavior are studied. Different forms of hyperchaotic complex Chen systems are constructed. We suppress the hyperchaotic behavior of our system via passive control method by using one complex controller. The hyperchaotic attractors of the new system are converted to its unstable trivial fixed point and tracked to its unstable non trivial fixed points and periodic orbits. Block diagrams of our system are designed by using Matlab/Simulink after and before the suppression process to ensure the validity of the analytical results.

1. Introduction

In 1982, Fowler introduced a complex Lorenz model, as a generalization of the real Lorenz, which is used to describe and simulate the physics of detuned lasers and thermal convection of liquid flows [1,2]. The electric field and atomic polarization amplitude in these systems are both complex quantities, whose real and imaginary parts can display chaotic dynamics [3]. Basic properties and chaotic synchronization of the complex Lorenz model are studied by Mahmoud et al. [4]. In recent years, beside the famous complex Lorenz model, several other such examples have been proposed in the literature, notably the so-called complex Chen and Lü systems, which may be thought to belong to the same class as the Lorenz equation [5–9], and references there in. These systems which involve complex variables are used to describe the physics of a detuned laser, rotating fluids, disk dynamos, electronic circuits and particle beam dynamics in high energy accelerators.

The chaotic complex Chen system is introduced in [5] as:

$$\begin{aligned} \dot{z_1} &= \alpha(z_2 - z_1), \\ \dot{z_2} &= (\gamma - \alpha)z_1 - z_1 z_3 + \gamma z_2, \\ \dot{z_3} &= 1/2(\bar{z_1} z_2 + z_1 \bar{z_2}) - \beta z_3, \end{aligned} \tag{1}$$

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where $\alpha,\ \gamma$ and β are positive (real) parameters, $z_1 = \eta_{11} + i\eta_{12},\ z_2 = \eta_{21} + i\eta_{22}$ and $z_3 = \eta_{31}$. Dots represent derivatives with respect to time, an "overbar" denotes complex conjugate variables and $i = \sqrt{-1}$.

In the last three decades several researchers have focused their attention on the study of hyperchaotic systems in many important fields in applied nonlinear sciences, e.g. laser physics, secure communications, nonlinear circuits, synchronization, control, neural networks and active wave propagation [10–13]. A hyperchaotic attractor is typically defined as chaotic behavior with at least two positive Lyapunov exponents. The minimal dimension for a (continuous) hyperchaotic system is 4 [14].

Many ideas have been employed to generate hyperchaotic systems with complex variables. Some of these are adding state feedback control, periodic forcing and linear controller to chaotic complex systems (e.g. [15–19] and references therein).

In this work we introduce a new hyperchaotic model with complex variables by adding a complex nonlinear term to the third equation of the chaotic complex Chen system (1) as:

$$\begin{aligned}
\dot{z_1} &= \alpha(z_2 - z_1),\\
\dot{z_2} &= (\gamma - \alpha)z_1 - z_1 z_3 + \gamma z_2,\\
\dot{z_3} &= 1/2(z_1\bar{z}_2 + \bar{z}_1 z_2) + iRe(z_1)Im(z_2) - \beta z_3,
\end{aligned} \tag{2}$$

where $\alpha,\ \gamma$ and β are positive (real) parameters of complex Chen model, $z_1 = \eta_{11} + i\eta_{12},\ z_2 = \eta_{21} + i\eta_{22}$ and $z_3 = \eta_{31} + i\eta_{32}$ are complex function, $Re(z_1)$ is the real part of z_1 equals η_{11} and $Im(z_2)$ is imaginary part of z_2 equals η_{22}. We call system (2) a hyperchaotic complex Chen system. All of the variables in system (2) are complex.

More recently however, some new kinds of synchronization have been proposed exclusively for complex dynamical systems. These new kinds were studied for several examples of chaotic and hyperchaotic complex nonlinear systems, seeking to synchronize both the modulus and phase of their oscillations [20,21]. In literature the chaotic and hyperchaotic complex systems have complex and real variables [4–9,15–19]. The motivation of this work is to introduce the first hyperchaotic nonlinear system with its all variables is complex. As is well known, there exist interesting cases of dynamical systems where the main variables participating in the dynamics are complex, for example, when amplitudes of electromagnetic fields and atomic polarization are involved [5]. Increasing the number of variables (or introducing complex variables) is also crucial in chaos synchronization used in secure communications, where one wishes to maximize the content and security of the transmitted information.

This paper is organized as follows: in the next section symmetry, invariance, fixed points and stability analysis of the trivial fixed points of (2) are discussed. The complex behavior of (2) is studied. Numerically the range of parameters values of the system at which hyperchaotic attractors exist is calculated based on the calculations of Lyapunov exponents. The signs of Lyapunov exponents provide a good classification of the dynamics of (2). The fractional Lyapunov dimension is calculated to hyperchaotic attractors of (2). Bifurcation analysis is used to demonstrate chaotic and hyperchaotic behaviors of (2). The block diagram of our system (2) is constructed by using Matlab/Simulink. Some Figures are presented to show our investigations. We construct different forms of hyperchaotic complex Chen systems in Section 3. In the fourth section the passive control method is used to suppression the hyperchaotic behavior of (2) to trivial fixed point, non trivial fixed points and periodic attractors. The block diagram of our system (2) after the suppression process is designed by using Matlab/Simulink to ensure the validity of the theoretical results. In the last section the main conclusions of our investigations are summarized.

2. Dynamical behaviors of system (2)

In this section we study the basic dynamical analysis of our new system (2). The real version of (2) reads:

$$\begin{aligned}
\dot{\eta}_{11} &= \alpha(\eta_{21} - \eta_{11}),\\
\dot{\eta}_{12} &= \alpha(\eta_{22} - \eta_{12}),\\
\dot{\eta}_{21} &= (\gamma - \alpha)\eta_{11} - \eta_{11}\eta_{31} + \eta_{12}\eta_{32} + \gamma\eta_{21},\\
\dot{\eta}_{22} &= (\gamma - \alpha)\eta_{12} - \eta_{12}\eta_{31} - \eta_{11}\eta_{32} + \gamma\eta_{22},\\
\dot{\eta}_{31} &= \eta_{11}\eta_{21} + \eta_{12}\eta_{22} - \beta\eta_{31},\\
\dot{\eta}_{32} &= \eta_{11}\eta_{22} - \beta\eta_{32}.
\end{aligned} \tag{3}$$

System (3) has the following basic dynamical properties:

2.1. Symmetry and invariance

In (3), we note that this system is invariant under the transformation:

$$(\eta_{11}, \eta_{12}, \eta_{21}, \eta_{22}, \eta_{31}, \eta_{32}) \to (-\eta_{11}, -\eta_{12}, -\eta_{21}, -\eta_{22}, \eta_{31}, \eta_{32}).$$

Therefore, if $(\eta_{11}, \eta_{12}, \eta_{21}, \eta_{22}, \eta_{31}, \eta_{32})$ is a solution of (3), then $(-\eta_{11}, -\eta_{12}, -\eta_{21}, -\eta_{22}, \eta_{31}, \eta_{32})$ is also a solution of the same system.

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