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Wave scattering by soft–hard three spaced waveguide

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ABSTRACT

In this work we consider the sound radiation of a fundamental plane wave mode from a semi-infinite soft–hard duct. This duct is symmetrically located inside an infinite duct. This infinite waveguide consist of soft and hard plates. The whole system constitutes a three spaced waveguide. A closed form solution is obtained by using eigenfunction expansion matching method. This particular problem has been solved previously by Rawlins in closed form but without numerical work. Here the numerical results for reflection coefficient are given when the lowest mode propagates out from the semi-infinite duct. At the end we give comparison to both methods.

1. Introduction

Unpleasant noise is an unwanted major pollutant in an industrialised society. The unwanted noise from exhaust system of transportation vehicles are major sources of noise pollution. One way to reduce noise is to design novel geometries, and the use of various sound absorbent materials for exhaust system. The scattering problems of acoustic waves by half plane satisfying soft, hard and impedance boundary conditions are important in the diffraction theory and have been investigated by many researchers (see [1–7]). The case of non-staggered half planes in three equal space were firstly studied by Jones [8]. His problem ended in 3×3 matrix Wiener Hopf equations. The equation is reduced first to a system of 2×2 matrix Wiener Hopf equations with a single scalar. In [8] an explicit solution and various properties of scattered field for a plane wave to the scattering problem are obtained. One should note that it is not always possible to solve problems involving 2×2 matrices using Wiener–Hopf technique because the original decomposition is not available in the literature.

Although some trifurcated waveguide problems has been solved by using standard scalar Wiener Hopf technique with the result of symmetry. For example, Rawlins [1], a mathematical model to predict noise in an exhaust system was proposed. This model resulted in Wiener Hopf problem having a matrix to be factorized.

Mahmood-ul-Hassan and Rawlins [9] have also used the Wiener–Hopf technique very successfully for solving two problems of radiation of sound in the trifurcated ducts. They have considered the compressible fluid in motion with constant fluid velocity. They have presented their solution in the form of contour integrals which are evaluated in terms of infinite series of modes. These infinite series of modes are propagating in the trifurcated waveguide. Numerical results have been presented for reflection field amplitude for particular values of the ratio of the fluid velocity and the speed of sound. Both particular models can be used as the possible practical exhaust splitter plates models.

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The present trifurcated waveguide problem has been solved by Rawlins. He published this problem as a technical report in [1] which was subsequently published as part of the paper [2]. Rawlins ended this problem in Wiener Hopf problem having non-trivial matrix. This matrix Wiener Hopf problem has been solved explicitly. He has presented closed form solution in [1,2] but has involved complicated factor/split functions. There is no numerical work for reflection coefficient in [1,2] which may be due to these complicated factors. In this paper, we solve this problem by using eigenfunction expansion matching method. This method is straightforward and easy as compared with the Wiener–Hopf equations technique. Graphical results are given in terms of reflection coefficient.

We divide our problem into four regions. The value of potential in different regions can be found by using separation of variables since the geometry is simple. The matched eigenfunction expansions [10] can then be used to obtained the solution. This method have been employed in many scattering problems (see [11–15]), and we use this method in this work. It should be noted that rapid convergence of this method is somewhat reduced for the sharp edges of the semi-infinite plates. At the end, we give analysis on the numerical results by plotting $|R|$ versus the wave number ka when the lowest mode is assumed to propagate. This would be helpful in modelling the exhaust practical system.

2. Three spaced waveguide problem

We consider the acoustic diffraction of a plane wave mode which propagates out of the open end of a semi-infinite duct. Fig. 1 shows the geometry of the problem. The geometry of the trifurcated waveguide problem is such that the semi-infinite duct consists of soft and hard plates. This semi-infinite duct is symmetrically located inside the infinite duct having soft and hard boundary conditions on the plates. The sound source field, which is located at $y = y_0,\ x = x_0\ (x_0 < 0,\ -a < y_0 < a)$ and propagate modes across the semi-infinite duct. We define the acoustic pressure of the potential function $\varphi(x, y, t)$ by

$$P = -\rho_o \frac{\partial \varphi}{\partial t},$$

where ρ_o is the density in the equilibrium state. We define the velocity by

$$\vec{u} = grad(\varphi).$$

The incident sound field is assumed to have time variation e^{-iwt} with the wave number $k = \frac{w}{c},\ w$ is angular frequency and c is the speed of sound.

We shall remove the time from the problem by writing

$$\varphi(x, y, t) = \mathrm{Re}[\phi(x, y)e^{-iwt}], \tag{1}$$

in the rest of the work. Now, by using (1) with the wave equation given by

$$\nabla^2\varphi = (1/c^2)\varphi_{tt}, \tag{2}$$

these two equations give

$$\nabla^2\phi(x, y) + k^2\phi(x, y) = 0, \tag{3}$$

which is the two dimensional Helmholtz equation to be solved for $\phi(x, y)$ in the trifurcated waveguide system with the boundary conditions:

$$\phi(x, y) = 0, \quad y = b, \quad -\infty < x < \infty; \tag{4}$$

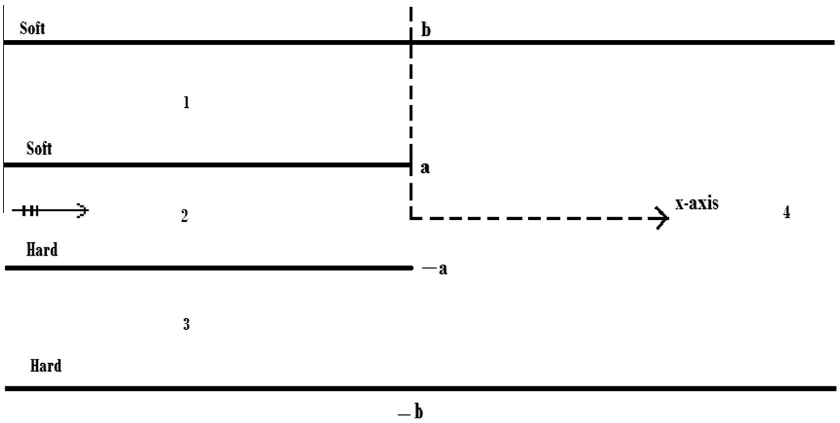

Fig. 1. Geometry of three spaced duct.