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A new modified Adomian decomposition method and its multistage form for solving nonlinear boundary value problems with Robin boundary conditions

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A B S T R A C T

In this paper we propose a new modified recursion scheme for the resolution of boundary value problems (BVPs) for second-order nonlinear ordinary differential equations with Robin boundary conditions by the Adomian decomposition method (ADM). Our modified recursion scheme does not incorporate any undetermined coefficients. We also develop the multistage ADM for BVPs encompassing more general boundary conditions, including Neumann boundary conditions.

1. Introduction

We propose a new resolution method for boundary value problems (BVPs) with Robin boundary conditions, including BVPs with mixed sets of boundary conditions, for nonlinear second-order differential equations by the Adomian decomposition method (ADM) [1–14]. This new approach is based on the Duan–Rach modified recursion scheme for the ADM [15], where we transform the original nonlinear BVP into an equivalent nonlinear Fredholm–Volterra integral equation for the solution before designing the recursion scheme. Our new algorithm for the solution of Robin BVPs subsumes the set of Dirichlet boundary conditions as well as mixed sets of Robin and Dirichlet, Robin and Neumann, Dirichlet and Robin, Dirichlet and Neumann, Neumann and Dirichlet, and Neumann and Robin boundary conditions.

Furthermore we develop a multistage ADM for BVPs through partitioning the domain into two, or more, subdomains, where we compute a separate series in each subdomain using our new modified recursion scheme for nonlinear BVPs. The sub-solutions are combined by applying the conditions of continuity at the interior boundary points in analogy to the multistage ADM for initial value problems (IVPs) [16–23].

We show how our multistage ADM for BVPs can easily treat nonlinear examples when the original series diverges over the specified domain. Another aim of the multistage ADM for BVPs is to solve nonlinear Neumann BVPs relying upon the key concept of converting the original BVP into two sub-BVPs, where each is subject to a mixed set of Neumann and Dirichlet boundary conditions.

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The ADM is a well-known systematic method for practical solution of linear or nonlinear and deterministic or stochastic operator equations, including ordinary differential equations, partial differential equations, integral equations, integro–differential equations, etc. [2–14,24]. Adomian's decomposition method is a powerful technique, which provides efficient algorithms for analytic approximate solutions and numeric simulations for real-world applications in the applied sciences and engineering. The ADM permits us to solve both nonlinear IVPs and BVPs [7,25–41] without unphysical restrictive assumptions such as required by linearization, perturbation, ad hoc assumptions, guessing the initial term or a set of basis functions, and so forth. Furthermore the ADM does not require the use of Green's functions which are not easily determined in most cases. A key notion is the Adomian polynomials, which are tailored to the particular nonlinearity to solve nonlinear differential equations.

Adomian and co-workers have solved nonlinear differential equations with a wide class of nonlinearities, including product [42], polynomials [43], exponential [44], trigonometric [45], hyperbolic [46], composite [47], negative-power [48], radical [49] and even decimal-power nonlinearities [50]. We find that the ADM solves nonlinear operator equations for any analytic nonlinearity, providing us with an easily computable, readily verifiable, rapidly convergent sequence of analytic approximate functions.

Several investigators including Cherruault and co-workers [51–53] among others have previously proved convergence of the Adomian decomposition series and the series of the Adomian polynomials. For example, Abdelrazec and Pelinovsky [54] have published a rigorous proof of convergence for the ADM under the aegis of the Cauchy–Kovalevskaya theorem for IVPs. Furthermore Agarwal [55] has provided the prerequisites for existence and uniqueness of solutions for BVPs, including higher order BVPs. A key concept is that the Adomian decomposition series is a computationally advantageous rearrangement of the Banach-space analog of the Taylor expansion series about the initial solution component function, which permits solution by recursion. A remarkable measure of success of the ADM is demonstrated by its widespread adoption and many adaptations to enhance computability for specific purposes, such as various modified recursion schemes beginning with those by Adomian and Rach [26,27,56], Wazwaz [57], Wazwaz and El-Sayed [58], Duan [59–61], and Duan and Rach [62,15]. The choice of decomposition is nonunique, which provides a valuable advantage to the analyst, permitting the freedom to design modified recursion schemes for ease of computation in realistic systems.

In Section 2, we present a brief review of the ADM for nonlinear IVPs and BVPs to provide a common ground for the sequel. In Section 3, we present a description of our new approach for solving nonlinear Robin BVPs. We develop a systematic algorithm that also encompasses nonlinear BVPs with the set of Dirichlet boundary conditions as well as mixed sets of Robin and Dirichlet, and Robin and Neumann boundary conditions. Furthermore, we propose the multistage modified decomposition solution of nonlinear BVPs. In Section 4, we first consider other inverse linear operators for computational advantage, then apply them to the cases of mixed sets of Dirichlet and Robin, Neumann and Robin, Dirichlet and Neumann, and Neumann and Dirichlet boundary conditions. In Section 5, we next investigate four expository numerical examples including a nonlinear BVP with a set of Neumann boundary conditions. In the sequel, we present our conclusions and summarize our findings.

2. Review of the Adomian decomposition method

We next review the salient features of the Adomian decomposition method in solving IVPs and BVPs for nonlinear deterministic differential equations. Consider the general nonlinear deterministic differential equation in Adomian's operator-theoretic form

$$Lu + Ru + Nu = g, \tag{2.1}$$

where g is the system input and u is the system output, and where L is the linear operator to be inverted, which usually is just the highest order differential operator, R is the linear remainder operator, and N is the nonlinear operator, which is assumed to be analytic. We remark that this choice of the linear operator is designed to yield an easily invertible operator with resulting trivial integrations. Furthermore we emphasize that the choice for L and concomitantly its inverse L^{-1} are determined by the particular equation to be solved, hence the choice is nonunique, e.g. for cases of differential equations with singular coefficients, we choose a different form for the linear operator. Generally we choose $L = \frac{d^p}{dx^p}(\cdot)$ for pth-order differential equations and thus its inverse follows as the p-fold definite integration $L^{-1}(\cdot) = \int_{a_1}^{x} \ldots \int_{a_p}^{x} (\cdot) dx \ldots dx$. For IVPs, all of the lower limits of integration a_k are equal, i.e. $a_k = a$, which is the initial point, while the values of a_k may differ according to the distinct number of boundary points of the specified BVP. Our overarching aim is to achieve easy-to-integrate series by the ADM for computing the solution approximations.

We first solve Eq. (2.1) for the linear term Lu

$$Lu = g - Ru - Nu. \tag{2.2}$$

Since L has been assumed to be invertible, we apply the inverse linear operator L^{-1} to both sides of Eq. (2.2)

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu. \tag{2.3}$$

By the definition of integral operators, we also have

$$L^{-1}Lu = u - \Phi, \tag{2.4}$$