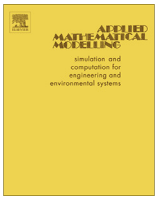
ARTICLE IN PRESS

Applied Mathematical Modelling xxx (2013) xxx–xxx

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm



Numerical investigations of supercavitation around blunt bodies of submarine shape

Zhi Shang *

*Institute of High Performance Computing, A*STAR, 1 Fusionopolis Way, #16-16 Connexis, Singapore 138632, Singapore*

A R T I C L E I N F O

Article history:
Received 19 May 2012
Received in revised form 15 March 2013
Accepted 8 April 2013
Available online xxxx

Keywords:
Cavitation
Supercavitation
Blunt body
High speed
Submarine

A B S T R A C T

Cavitation occurs when liquid is subjected to rapid changes of pressure. If the local pressure is lower than the liquid saturation pressure, the liquid changes its phase into vapor. A fast moving solid body underwater will reduce the local pressure around the body. Under certain conditions, the local pressure can be lower than the water saturation pressure, accordingly causing cavitation. If the velocity of the moving body increases further, a supercavitation will occur. In this paper, the cavitation and supercavitation were studied by the SST k–ω turbulence model combined with a finite-rate mass transfer modeling under the mixture assumption. The validations of these models were performed through comparisons between numerical simulations and experiments. After the validations, the supercavitation generated by a high speed moving body, which can be described as a large bubble, was studied. The studies were on several blunt bodies, such as a submarine hull shape, a submarine hull shape with appendages and a full submarine shape with sail and appendages. It was found that the naked submarine hull is difficult to have the supercavitation covering the whole body. The sail and appendages can help supercavitation occurs earlier. Furthermore the effects of the supercavitation on the submarine and generation mechanism of the supercavitation is discussed based on the simulations.

1. Introduction

Cavitation is a complex physical phenomenon. It occurs when the local liquid pressure is lower than its saturated vapor pressure P_{sat} [1]. The device handling liquids might be subject to cavitation. Normally when cavitation occurs, it can affect the performance of turbo-machinery, resulting in a drop in head and efficiency of pumps, thereby decreasing the power output and efficiency of hydro-turbines [2]. However, a supercavitation can envelope the moving body inside a large continuous cavity. It can reduce the drag of an underwater moving body, thus enabling it to move at a high speed under water [3]. Understanding the phenomenon of cavitation is therefore important for the design of the devices handling water.

Cavitation is a 3-dimensional (3D), unsteady and discontinuous or periodic phenomenon of formation, growth and rapid collapse of bubbles [2]. Due to the natural complexity of cavitation, it is neither reliably assessable nor fully understood yet. Through experimental observations using modern diagnostics, some significant insights have been discovered. However, despite these advances there remains much to be understood and cavitation is still an area of ongoing research [4].

With the advances in computer power and numerical simulation techniques, it is now feasible to study 3D flow issues through computational fluid dynamics (CFD). Nowadays CFD is a useful complement to experimental data in order to develop improved design guidelines in many areas [5–7]. CFD can deal with complex fluid flow issues through direct numerical simulation (DNS) or modeling of complicated physical phenomena. Cavitation includes the phase change from liquid to

---

* Tel.: +65 64191547; fax: +65 64674350.
E-mail addresses: shangzhi@tsinghua.org.cn, shangz@ihpc.a-star.edu.sg.

0307-904X/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.apm.2013.04.009

Please cite this article in press as: Z. Shang, Numerical investigations of supercavitation around blunt bodies of submarine shape, Appl. Math. Modell. (2013), http://dx.doi.org/10.1016/j.apm.2013.04.009

vapor through micro nucleation to the macro bubble [1]. A large number of computational grids and small time steps have to be employed if the numerical simulation is carried out. It is therefore difficult to use DNS to do the simulation from microscopic level to macroscopic level. The mathematical models describing the cavitation mechanisms based on certain assumptions can avoid such difficulties.

In this paper, the SST k–ω turbulence model [8] is employed to simulate the cavitation caused by a fast moving body under water. During the simulations, the transport equations of the two-phase mixture flow and the local volume fraction of vapor are solved through a finite rate mass transfer model of the vaporization and condensation processes. After the validation through comparisons of numerical simulations with experiments of a hemispherical head shape [9], the supercavitation around several blunt bodies, such as a submarine hull shape, a submarine hull shape with appendages and a full submarine shape with sail and appendages, which are based on a DARPA submarine model [10], was studied. The study could provide some useful information of supercavitation for the submarine moving at high speed under water.

2. Computational modeling of cavitation flows

When cavitation occurs the liquid changes its phase into vapor in certain flow regions where local pressure is very low due to high local velocities, which can be presented by Bernoulli equation along the streamline [1]. The process at the beginning of cavitation is called cavitation inception which can be described through the comparisons between the cavitation number and the pressure coefficient.

Usually, the cavitation number σ is defined by [1]:

$$\sigma = \frac{p_{ref} - p_{sat}}{\frac{1}{2}\rho_l U^2}, \tag{1}$$

where p_{ref} is the reference pressure of the liquid, p_{sat} the saturated vapor pressure of the liquid, ρ_l is the liquid density and U is a characteristic velocity of the flow.

The non-dimensional pressure coefficient C_p is defined through the local static pressure p_{loc} as:

$$C_p = -\frac{p_{loc} - p_{ref}}{\frac{1}{2}\rho_l U^2}. \tag{2}$$

When $\sigma \leqslant C_p$, cavitation occurs. Normally supercavitation occurs under $\sigma \leqslant 0.1$ [3].

2.1. Multiphase flow modeling

Due to the phase changes from liquid to vapor happens under cavitations, a multiphase flow model has to be employed to describe the flow. Usually the two-phase mixture governing equation is employed to describe the multiphase flow for cavitation.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial t} = 0, \tag{3}$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_i} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] + \frac{\partial}{\partial x_j}(\tau_{tij}) + \rho g_i + f_\sigma, \tag{4}$$

where u_i and p are the velocity and pressure, respectively. The mixture density ρ and viscosity μ are defined by

$$\rho = \alpha_v \rho_v + \alpha_l \rho_l, \tag{5}$$

$$\mu = \alpha_v \mu_v + \alpha_l \mu_l, \tag{6}$$

in which $\alpha_v + \alpha_l = 1$, ρ_v and μ_v are vapor density and viscosity, ρ_l and μ_l are liquid density and viscosity, a_v and a_l are the local vapor and liquid volume fraction, respectively. The local liquid volume fraction α_l is governed by

$$\frac{\partial \alpha_l}{\partial t} + \frac{\partial(\alpha_l u_i)}{\partial x_i} = -\frac{\dot{m}}{\rho_l}, \tag{7}$$

where $\dot{m}$ is the mass transfer rate between the phases.

In the above set of equations, some important terms such as Reynolds stress τ_{tij}, the surface tension f_σ and the mass transfer rate $\dot{m}$ need to be modeled.

2.2. Turbulence modeling

The Reynolds stress can be modeled through Boussinesq hypothesis [11] as the following equation:

$$\tau_{tij} = \mu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\left(\rho k + \mu_t \frac{\partial u_k}{\partial x_k}\right)\delta_{ij}, \tag{8}$$

Please cite this article in press as: Z. Shang, Numerical investigations of supercavitation around blunt bodies of submarine shape, Appl. Math. Modell. (2013), http://dx.doi.org/10.1016/j.apm.2013.04.009