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Some scheduling problems with time-dependent learning effect and deteriorating jobs

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ABSTRACT

Although machine scheduling problems with learning and deteriorating effects consideration have received increasing attention in the recent years, most studies have seldom considered the two phenomena simultaneously. However, learning and deteriorating effects might co-exist in many realistic scheduling situations. Thus, in this article, a model which takes the effects of time-dependent learning and deterioration simultaneously is proposed and applied into some scheduling problems. Under the proposed model, the processing time of a job is determined by a function of its corresponding starting time and positional sequence in each machine. We show that some single machine and flowshop scheduling problems are polynomially solvable with the certain performance measures such as makespan, total completion time, and weighted completion time.

1. Introduction

The effects of learning and deterioration have been widely seen in real-life situations such as in financial management, steel production scheduling, resource allocation, where any delay in tackling a task may result in an increasing/decreasing effort (time, cost, etc.) to accomplish the task [11]. Those phenomena concerned in the scheduling might be reflected into the job processing that expected processing time of a job increases (or decreases) linearly or it is piecewised linearly on its starting time. In the classical scheduling problems, the processing times of jobs are assumed to be fixed values. However, in many realistic situations, the actual processing time of a job can be more or less than its normal processing time if it is scheduled later. The phenomenon where the actual processing time of a job is shorter or longer than its corresponding processing time if the job is scheduled later, rather than scheduled earlier in the sequence is known as the "learning effect or deterioration" in literature.

Some articles introducing a learning or deteriorating effect model into a scheduling problem have been investigated. In these articles, Koulamas and Kyparisis [1] developed a sum-of-processing-time-based learning model for single-machine and two-machine scheduling problems. The results proved that both problems are polynomially solvable and that the shortest processing time (SPT) sequence is the optimal schedule for the objective of minimizing the makespan and the total completion time. Mosheiov and Sidney [4] extended the job-dependent learning curve, and proposed a more realistic learning model for a single or an unrelated parallel machine scheduling problem. The authors proved that a single machine problem with the minimization of makespan and total flow time objectives remains polynomially solvable, and an unrelated parallel machine scheduling problem with the minimization of total flow time objective is, as well. Wang [9] considered a single-machine scheduling problem with a sum-of-processing-times-based learning effect and proved that the shortest processing time (SPT) rule can provide an optimum schedule for the minimization of the sum of square completion time objective. Lee

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and Wu [17] proposed a learning effect for a single machine scheduling problem with minimization of makespan or total completion time objectives. The authors proved the problem remains polynomially solvable under their proposed learning effect model. Kuo and Yang [14,15] considered a single-machine scheduling problem with a time-dependent learning effect. In their study, the time-dependent learning effect of a job is assumed to be a function of the total normal processing time of the jobs scheduled in front of it. Their results show that the SPT sequence is optimal with the minimization of the total completion time objective. Kuo and Yang [16] also proposed a time-dependent learning effect model for a single-machine group scheduling problem. They showed that a single-machine group scheduling problem with a time-dependent learning effect remains polynomially solvable for the objective of minimizing the makespan and total completion time. Browne and Yechiali [18] introduced a scheduling problem with deteriorating jobs in which the processing time of a job is a function of its starting time. The authors analyze the effects of different deterioration schemes and derive optimal scheduling policies with the minimization of the makespan.

Bai et al. [10] proposed a single-machine group scheduling problems with effects of learning and deterioration at the same time. The effects of learning and deterioration mean that the group setup times are general linear functions of their starting times and the jobs in the same group have general position-dependent and time-dependent learning effects. In his model, the actual setup time of group G_i is

$$s_i = a_i + b_i t, \quad i = 1, 2, \ldots, m,$$

where a_i is the basic setup time of group G_i, b_i is the setup deterioration rate of group G_i and t the start time to processing group G_i. The actual processing time of J_{ij} in group G_i is scheduled in the rth position is presented as

$$p^A_{ijr} = p_{ij} f_i\left(\sum_{l=1}^{r-1} p_{i[l]}\right) g_i(r), \quad i = 1, 2, \ldots, m; \quad r, j = 1, 2, \ldots, n_i,$$

where p_{ij} is the basic processing time of job J_{ij}; $\sum_{l=1}^{0} p_{[l]} := 0, f : [0, +\infty) \to [1, +\infty)$ is a differentiable non-increasing function with f' is non-decreasing on $[0, +\infty)$ and $f(0) = 1$, and $g : [1, +\infty) \to [0, +\infty)$ is a non-decreasing function with $g(1) = 1$. $\alpha_i \leqslant 0$ is a constant learning index of group G_i. The author showed that the problem with the minimization of objectives such as makespan and the sum of completion time can be optimally solved under the proposed model. Lee [13] proposed two deterioration or learning effects models for a single machine scheduling problem. In that article, in the first model, the actual processing time for a job J_i presented as $p_{ir} = \alpha_i t r^a$, when J_i starts at time t and is scheduled in position r in a sequence; where α_i is the deterioration rate, $a \leqslant 0$ is the learning rate and $t \geqslant t_0$ is the starting time of job J_i. The author showed that the optimal schedule can be obtained by the smallest deterioration rate (SDR) rule if the objective is to minimize makespan, flow time, or the sum of the lateness, respectively. In the second model, $p_{ir} = (p_0 + \alpha_i t) r^a$, where p_0 is the basic processing time, the author only gave counterexamples to demonstrate that the SDR rule does not provide the optimal solution for those objectives. Wang [8] considered a single-machine scheduling problems with the effects of learning and deterioration. In his model, the actual processing time of a job J_i is presented as $p_{ir} = a_i(\alpha(t) + \beta r^a)$, where $\alpha(t)$ $(\alpha(0) \geqslant 0)$ is an increasing (deterioration) function and weight parameter $\beta \geqslant 0$. The author showed that the problem with the minimization of objectives such as makespan, total completion time, and the sum of completion time squares can be optimally solved by the shortest processing time (SPT) rule.

The word "no-idle" means no machine is allowed to have an idle time between processing any two operations, i.e., each machine, once it commences its work, has to process all operations assigned to it without any interruption. Such a restriction may be very natural in some real life situations. For the circumstance, Wang and Xia [7] address an instance in order to explain a real-life situation on a no-idle machine, that is, where machines represent expensive pieces of equipment that have to be rented only for the duration between the start of the first operation and completion of the last one. Yang and Kuo [3] considered some scheduling problems with deteriorating jobs and learning effects. They showed that the problems with the minimization of objectives such as the makespan, the total completion times, and the total absolute differences in completion times can be solved in polynomial time. Finally, they also show that several special cases of flow shop scheduling problems remain polynomially solvable under the proposed model. Cheng et al. [11,12] addresses the permutation flow shop scheduling problems with a learning effect (or deteriorating jobs) on no-idle dominant machines. They showed that the problems with a minimized makespan or total completion time can be solved in polynomial time. Previous work on flow shop scheduling in an environment of a series of dominating machines can also be found in Ho and Gupta [6], and Xiang et al. [19].

2. Single-machine scheduling problems

2.1. Notation

In this section, the notation is defined and used throughout the single machine scheduling problems.

a: a learning effect index $(a \geqslant 1)$
b: a learning effect index $(0 < b < 1)$
s: a deterioration rate $(s \geqslant 0)$