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Closed-form compliance equations of filleted V-shaped flexure hinges for compliant mechanism design

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ABSTRACT

This paper presents the closed-form compliance equations for the filleted V-shaped flexure hinges. The in-plane and out-of-plane compliances of the flexure hinges are developed based on the Castigliano's second theorem. The accuracy of motion, denoted by the midpoint compliance of the flexure hinges, is also derived for optimized geometric design. The influences of the geometric parameters on the characteristics of the flexure hinges are investigated. It is noted that the filleted V-shaped flexure hinges have diverse ranges of compliance corresponding to different filleted radius R and angle θ. These types of hinges can provide both higher and lower stiffnesses than circular flexure hinges. This makes filleted V-shaped flexure hinges very useful for wide potential applications with different requirements. The finite element analysis is used to verify the established closed-form compliance equations for these filleted V-shaped flexure hinges.

1. Introduction

Flexure hinges are commonly utilized in the compliant mechanisms for applications in micro-/nano-instruments, machines and systems such as scanning tunnel microscope, X-ray lithography, mask alignment, and micro-manufacturing [1–8], where high positioning accuracy and resolution are the necessary and crucial requirements to fulfil specified tasks. Flexure-based mechanisms can overcome the shortcomings such as stiction, friction, and backlash which generally exist in the conventional mechanisms with sliding and rolling bearings. Thus, flexure-based mechanisms are capable of smooth motion, free of friction and lubrication [9–14]. The positioning accuracy of flexure-based mechanisms can be further improved by utilizing laser-interferometry-based sensing technique for independent position and displacement measurement and tracking [15,16]. In order to obtain high performance flexure-based mechanisms, CAD (computer aided design) methodology is usually utilized to conduct the mechanical design and optimization, and to develop the fixture structure using 3-2-1 locating and clamping technique for holding such monolithic mechanisms [17,18]. With the aid of CNC (computer numerical control) and WEDM (wire electrical discharge machining) techniques, flexure hinges can be monolithically manufactured with other links of the entire mechanism. This makes it possible to achieve high machining accuracy and eliminate the errors of assembly. Furthermore, the geometric and dimensional tolerances can be exactly controlled, and the positioning accuracy of the entire system is improved. In the precision positioning, the flexure-based mechanisms are also used to enlarge the small displacement of the actuator or the small output force to driving moving platform [19–24]. Unlike conventional revolute joints, flexure hinges have finite stiffness in the output direction. Therefore, the rotational center will offset when the flexure hinges generate output displacement. This is the negative aspect of using flexure-based mechanisms [25,26]. In order to achieve efficient compliant mechanisms for applications in nano-manipulation, it is important to correctly choose the geometric parameters and to predict and optimise the performance of flexure hinges.

In flexure-based mechanisms, flexure hinges are generally made of the rectangular blank removing two symmetric cut-outs with the profiles of circular, corner-filleted, and elliptical profiles as shown in Fig. 1. These kinds of flexure hinges have low rotational stiffness about one axis providing displacement and high stiffness in other degrees of freedom. The analytical solutions for compliance in these kinds of flexure hinges have been investigated in the past. Paros and Weisbord [27] presented the closed-form equations and curves of the circular flexure hinges for both symmetric single-axis and two-axis configurations based on the theory of mechanics of materials. The

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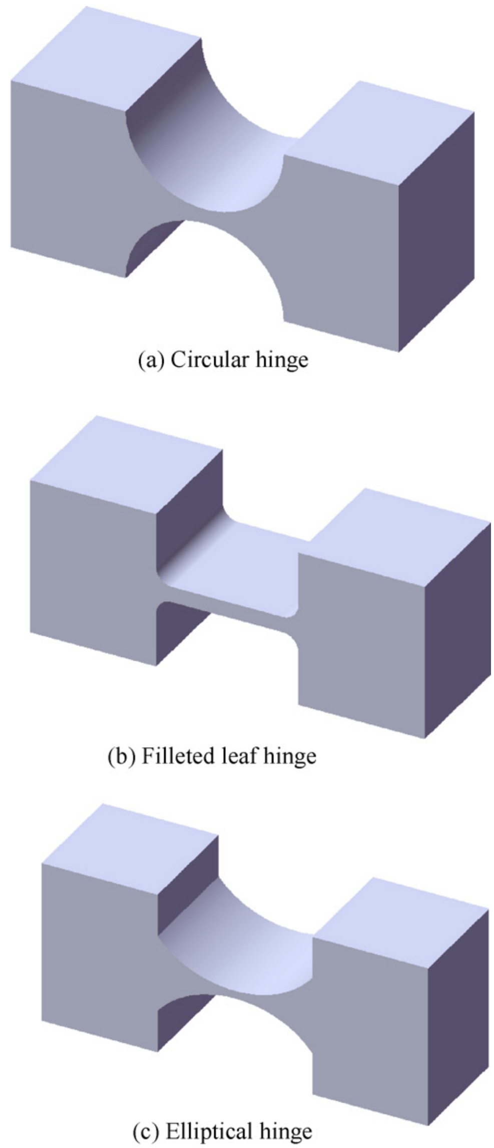

Fig. 1. Three traditional flexure hinges.

angular and linear compliances were developed for bending and axial loads of the in-plane and out-of-plane, respectively. For each case, the solutions were expressed in both exact and simplified forms. Using the same method, Smith et al. [28] developed the closed-form equations for symmetric single-axis semi-elliptical flexure hinges. It was noted that the performance of the elliptical flexure hinges were within the range constrained by the circular flexure hinges and the leaf hinges in terms of compliance. The finite element analysis and experimental tests were carried out to verify model predictions over a range of the typical geometries for many flexure hinge designs. Using finite element analysis, Xu and King [29] investigated the performance of the circular, corner-filleted and elliptical flexure hinges in terms of motion, stiffness and stress concentrations. Compared with the corner-filleted flexure hinges, the elliptical flexure hinges have less stress when the deflections are the same. The right circular flexure hinges have the highest stiffness, and the corner-filleted flexure hinges have the lowest stiffness. Lobontiu et al. [30] and Lobontiu and Garcia [31] derived the closed-form compliance equations for corner-filleted flexure hinges using Castigliano's second theorem. Similar to elliptical flexure hinges, corner-filleted flexure hinges also range within the domain confined by the right circular flexure hinges and the leaf flexure hinges in terms of compliance. The finite element analysis and experimental tests were used to confirm the model predictions. It was noted that the corner-filleted flexure hinges are more compliant and induce lower stress, however these are less accurate in rotation compared with the right circular flexure hinges. Utilizing a similar approach, Lobontiu et al. [32,33] developed the closed-form equations for a symmetric conic-section such as circular, elliptic, parabolic, and hyperbolic flexure hinges. The in-plane and out-of-plane compliance equations of the flexure hinges as well as the motion accuracy were derived for specific conic profiles. The finite element analysis and experimental tests were utilized to examine the model predictions. Wu and Zhou [34] introduced a different intermediate variable and presented the concise closed-form equations for circular flexure hinges. The comparison with equations developed by Paros was carried out. Tseytlin [35] presented the closed-form compliance equations for monolithic flexure hinges with circular and elliptical sections. The inverse conformal mapping of circular approximating contour was utilized to derive the analytical solutions. The predictions of the developed models were likely to be much closer to the finite element analysis and experimental data. Schotborgh et al. [36] described the dimensionless design graphs for three flexure hinges