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Unsteady motion of a spherical bubble in a complex fluid: Mathematical modelling and simulation

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ABSTRACT

The nonlinear response of an oscillatory bubble in a complex fluid is studied. The bubble is immersed in a Newtonian liquid, which may have a dilute volume fraction of anisotropic additives such as fibers or few ppm of macromolecules. The constitutive equation for the fluid is based on a Maxwell model with an extensional viscosity for the viscous contribution. The model is considered new in the study of bubble dynamics in complex fluids. The numerical computation solves a system of three first order ordinary differential equations, including the one associated with the solution of the convolution integral, using a fifth order Runge–Kutta scheme with appropriated time steps. Asymptotic solutions of governing equation are developed for small values of the pressure forcing amplitude and for small values of the elastic parameter. A study of the bubble collapse radius is also presented. We compare the results predicted by our model with other model in the literature and a good agreement is observed. The calculated asymptotic solutions are also used to test the results of the numerical simulations. In addition, the orientation of the additives is considered. The angular probability density function is assumed to be a normal distribution. The results show that the model based on the fully aligned additives with the radial direction overestimates the tendency of the additives to stabilize the bubble motion, since the effect of extensional viscosity occurs due to the particle resistance to the movement throughout its longitudinal direction.

1. Introduction

A single bubble behaves like a non-linear oscillator when immersed in a fluid undergoing external acoustic pressure field. Bubble dynamics has been a subject of several studies in different areas. For instance, in sonochemistry [1], in protein folding [2] or in the biomedical fields [3]: the ultrasound-induced cavitation is used for safe and efficient drug delivery, as well as for chemotherapy treatment [4,5]. Cavitation is also used to induce comminution of kidney stones [6]. The dynamical response of a spherical oscillating bubble in a Newtonian media largely appears in literature, nevertheless its behavior when immersed in a non-Newtonian fluid is far from complete. Ting [7] and Chahine and Fruman [8] observed that a fluid composed by polymer solution can reduce the collapse phenomenon. The non-Newtonian properties of the fluid may therefore contribute to the reduction of noise and cavitation damage as observed by Brujan [9]. Although different models on viscoelastic fluids have been recently used to explore bubble oscillations [10–12], even under the effect of a magnetic field [30] the influence of rheological properties of the host fluid, such as the fluid elasticity, still leaves open questions.

We propose a study of a spherical oscillating bubble immersed in a viscoelastic fluid. In this work, the ambient fluid is characterized as a substance composed of a Newtonian liquid and a dilute volume fraction of additives as long fibers or

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few ppm of macromolecules. As in a previous work by the authors [13], a Maxwell model with extensional viscosity has been also used for describing the constitutive equation for the fluid, which involves a convolution or memory integral. The model assumes a system with three non-linear first order equations, including an ordinary differential equation for the convolution integral, obtained by a similar procedure commonly used for different flows involving non-Newtonian fluid with memory [14]. For a more realistic description of the bubble dynamics, we take now into consideration the orientation of the additives. The average of the additives orientation is denoted as a constant S_0 and is defined in terms of the angular probability density function, assumed to be a normal distribution. For $S_0 = 1$, the additives are fully aligned additives with the radial direction, whereas $S_0 \cong 0.1$ gives an orientation condition perpendicular to the radial direction. The results show that fully oriented particles ($S_0 = 1$) as considered in [13] overestimates the tendency of the additives to stabilize the bubble motion, since the effect of extensional viscosity occurs due to the particle resistance to the movement throughout its longitudinal direction. The numerical method for solving the equations is explained in details, which includes the code validation and the perturbation method for the governing equation in terms of the pressure amplitude. We show a study to predict the bubble collapse radius, where an asymptotic theory for the bubble minimum radius is proposed. In addition, an asymptotic solution of the bubble dynamics for small values of the Deborah number (i.e. the elastic parameter) is also presented.

2. Mathematical formulation

Consider a spherical bubble immersed in a Newtonian incompressible fluid of viscosity μ and density ρ. The ambient fluid contains a volume fraction of anisotropic particles of length ℓ and diameter a, which present a high aspect ratio, $\ell/a \gg 1$. The inner side of the bubble is composed by a mixture of contaminant gas (which develops a polytropic process) and liquid vapor. This mixture acts like an energy cushion of the liquid while the bubble contracts. We assume that the bubble develops only radial motions due to superficial tension, simplifying our analysis to an unidimensional motion, maintaining its spherical shape. Therefore, factors like pressure gradients in the liquid, presence of gravitational field or surfactants are neglected. Although some studies deal with mass transfer in the liquid–gas interface of an oscillating bubble (e.g. [15]), it is not considered in the present work as it has relevant dynamical effects at very low ambient pressure and for great portion of vapor in the mixture inside the bubble [16]. Furthermore, if one takes into account mass flux, the non-sphericity [17] would also be an important factor as a nonuniform dynamic surface stress arises from the motion [18]. The pressure inside the bubble is supposed to be spatially uniform and non-equilibrium effects in the collapse instant are not taken into account. Heat conduction through the bubble interface may influence the bubble dynamics very strongly [19]. However, for an equilibrium vapor density relatively small the isothermal process here adopted is valid [20].

2.1. Governing equations and boundary conditions

The general governing equations for the motion of an incompressible fluid are given by the continuity equation and the momentum equation, as follows

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = \nabla \cdot \mathbf{\Sigma}, \tag{2}$$

where $\mathbf{\Sigma} = -p\mathbf{I} + 2\mu\mathbf{D} + \boldsymbol{\sigma}$ is the bulk stress tensor for a statistically homogeneous suspension, written in terms of the pressure field p, the identity tensor $\mathbf{I}$, the rate of strain tensor $\mathbf{D} = 1/2(\nabla u + (\nabla u)^T)$ and the extra stress tensor due to the presence of additives, $\boldsymbol{\sigma}$. The term $\mathbf{u}$ is the Eulerian velocity field. We assume that the bubble develops only radial motions due to surface tension (i.e. clean bubble), simplifying our analysis to an unidimensional motion, maintaining its spherical shape. In particular, the examined unsteady one-dimensional flow involving inertia and a nonlinear complex liquid is characterized by a strong nonlinear dynamics.

The boundary conditions on the bubble surface are specified as following. Consider fluid 1 in the bubble side and fluid 2 in the complex liquid side. The interface between the fluids is S and $\mathbf{n}$ is its unit normal vector. The interfacial tension coefficient is denoted by $\hat{\sigma}$. First, the velocity is continuous at the surface S, i.e.

$$\mathbf{u}_1 = \mathbf{u}_2 \quad \text{on} \quad S. \tag{3}$$

However, the stress is not continuous on the interface and, in general on S, the traction is given by

$$\mathbf{n} \cdot \|\mathbf{\Sigma}_2 - \mathbf{\Sigma}_1\| = \Delta\mathbf{f}, \tag{4}$$

where the traction is expressed as a sum of a normal and tangential components

$$\Delta\mathbf{f} = (\mathbf{n} \cdot \Delta\mathbf{f})\mathbf{n} + (\mathbf{I} - \mathbf{nn}) \cdot \Delta\mathbf{f}, \tag{5}$$

on the surface S. Defining $\mathbf{I}^s = (\mathbf{I} - \mathbf{nn})$ as being the identity surface unity tensor, we can write $\Delta\mathbf{f} = \mathbf{n}\Delta f^n + \mathbf{I}^s \cdot \Delta\mathbf{f}$. In this work, the constitutive equation for the bubble interface given by $\Delta\mathbf{f}$ assumes an isotropic membrane described by the following stress tensor [21].

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