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A simple four-unknown refined theory for bending analysis of functionally graded plates

Ashraf M. Zenkour *

Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia
Department of Mathematics, Faculty of Science, Kafrelsheikh University, Kafr El-Sheikh 33516, Egypt

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ABSTRACT

In the present paper, a refined trigonometric higher-order plate theory is simply derived, which satisfies the free surface conditions. Moreover, the number of unknowns of this theory is the least one comparing with other shear theories. The effects of transverse shear strains as well as the transverse normal strain are taken into account. The number of unknown functions involved in the present theory is only four as against six or more in case of other shear and normal deformation theories. The bending response of FG rectangular plates is presented. A comparison with the corresponding results is made to check the accuracy and efficiency of the present theory. Additional results for all displacements and stresses are investigated through-the-thickness of the FG rectangular plate.

1. Introduction

The continuity of properties of functionally graded materials (FGMs) may be achieved by gradually changing the volume fraction of the constituent materials, usually in the thickness direction only. This eliminates interface problems of composite materials and thus the stress distributions are smooth. In fact, FGMs have gained considerable attention as a potential structural material for future high-speed spacecraft and power generation industries.

In the simplest FG plates, two different material ingredients change gradually from one to the other. The most familiar FG plate is compositionally graded from a refractory ceramic as a bottom surface to a metal as a top surface or vice versa. Typically, FG plates are made from a mixture of ceramic and metal or a combination of different materials. The ceramic in an FGM offers thermal barrier effects and protects the metal from corrosion and oxidation, and the FGM is toughened and strengthened by the metallic composition. FG plates and shells are now developed for general use as structural elements in extremely high temperature environments and different applications.

A simplified shear and normal deformation plate theory is presented here for the bending of FG plates. For studying the responses of such plates, the so called equivalent single-layer plate theory is sufficient. This theory contains, for example, the classical (CPT), first-order (FPT), higher-order (HPT), and other shear deformation theories. The CPT ignores the transverse shear deformation and gives reliable results only for thin structures. The FPT is simple to implement and applied for both thick and thin laminated plates and gives acceptable results but depends on a shear correction factor which is hard to find as it depends on many parameters. However, there is no need of shear correction factors when using HPT or other shear deformation plate theories but governing equations are more complicated than those of the FSDT. In fact the HPT and other refined shear deformation theories give more accurate and stable transverse shear stresses.

* Address: Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia.
E-mail address: zenkour@gmail.com

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Just four unknown displacement functions are used in the present theory against six or more unknown displacement functions used in the corresponding ones. The effects due to transverse shear and normal deformations are both included. The theory does not require shear correction factors since the displacement components are expressed by trigonometric series representation through the plate thickness to develop a two-dimensional theory and gives rise to transverse shear stress variation such that the transverse shear stresses vary parabolically across the thickness satisfying shear stress free surface conditions. The numerical examples show that all stresses can be accurately calculated by the direct constitutive equation approach. The effectiveness of the present theory is demonstrated and results are compared with the corresponding FGM solution.

2. A simplified four-unknown plate theory

The displacements u_i of a material point in a FGM plate are assumed with the aid of the refined shear deformation plate theory (RPT) as [1–6]:

$$\left.\begin{aligned} u_1 &= u(x,y) - z\frac{\partial w}{\partial x} + f(z)\psi_1(x,y),\\ u_2 &= v(x,y) - z\frac{\partial w}{\partial y} + f(z)\psi_2(x,y),\\ u_3 &= w(x,y) + g(z)\varphi(x,y). \end{aligned}\right\} \tag{1}$$

In Eq. (1), u, v, and w are the displacements of the middle surface along the axes x, y and z, respectively, and ψ_1 and ψ_2 are the rotations about the y and x axes and account for the effect of transverse shear strains. An additional displacement φ accounts for the effect of normal stress is included. The coefficient of ψ_1 and ψ_2 is given by an odd function f of z while the coefficient of φ is given by an even function g of z. The displacement field of the classical plate theory is given by setting $f(z) = g(z) = 0$ [7], while that of the first order plate theory is given by setting $f(z) = z$ and $g(z) = 0$ [8–10].

The displacement field given in Eq. (1) with $g(z) = 0$ is presented by Touratier [11] for composite plates and it is extended to FG plates by Zenkour [5]. Soldatos [12] has presented a similar transverse shear deformation theory for homogeneous monoclinic plates. In recent years, many investigators have used the same displacement field with the same or different forms of the functions $f(z)$ and $g(z)$. Table 1 shows different forms of the particularity $f(z)$ and the transverse displacement u_3 of each displacement field based higher-order theory (HPT).

In most shear deformation theories, typically FG plates have been analyzed neglecting the thickness stretching ε_z, being the transverse displacement considered independent by thickness coordinates. Some recent work on the analysis of FG plates was presented. Karama et al. [14] have studied the mechanical behavior of laminated composite beam by new multi-layered laminated composite structures model with transverse shear stress continuity. Ferreira et al. [15] have used a trigonometric shear deformation theory for modeling symmetric composite plates discretized by a meshless method based on global multi-quadric radial basis functions. Jun and Hongxing [23] have developed the exact dynamic stiffness matrix of a uniform laminated composite beam based on trigonometric shear deformation theory as given in [14]. Aydogdu [16] has proposed a new higher-order shear deformable plate theory for bending, vibration and buckling analysis of composite laminates. Mantari et al. [17–19] have presented an analytical solution to the static analysis of FG plates, sandwich and composite plates, using recently developed higher-order shear deformation theories.

The effect of thickness stretching in FG plates has been investigated by Carrera et al. [24], using finite elements. Neves et al. [20–22] have presented an original hyperbolic sine shear deformation theory for the bending and free vibration analysis of FG plates. The in-plane displacements are given as in Eq. (1) while the transverse displacement is given by

Table 1
Different forms of the particularity $f(z)$ and the transverse displacement u_3 of a HPT.

HPT	$f(z)$	u_3
Touratier [11]	$\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$	$w(x,y)$
Soldatos [12]	$h\sinh\left(\frac{z}{h}\right) - z\cosh\left(\frac{1}{2}\right)$	$w(x,y)$
Reddy [13]	$z\left[1-\frac{4}{3}\left(\frac{z}{h}\right)^2\right]$	$w(x,y)$
Karama et al. [14]	ze^{-2z^2/h^2}	$w(x,y)$
Ferreira et al. [15]	$\sin\left(\frac{\pi z}{h}\right)$	$w(x,y)$
Aydogdu [16]	$zm^{\frac{-2z^2/h^2}{\ln m}}, m>0$	$w(x,y)$
Mantari et al. [17]	$zm^{-2z^2/h^2}, m>0$	$w(x,y)$
Mantari et al. [18]	$\tan(mz) - mz\sec^2\left(\frac{mh}{2}\right)$	$w(x,y)$
Mantari et al. [19]	$\sin\left(\frac{\pi z}{h}\right)e^{m\cos\left(\frac{\pi z}{h}\right)} + \frac{m\pi z}{h}$	$w(x,y)$
Zenkour [1–5]	$\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$	$w(x,y)$
Zenkour [6]	$\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$	$w+\cos\left(\frac{\pi z}{h}\right)\varphi$
Neves et al. [20,21]	$\sin\left(\frac{\pi z}{h}\right)$	$w+z\varphi_1+z^2\varphi_2$
Neves et al. [22]	$\sinh\left(\frac{\pi z}{h}\right)$	$w+z\varphi_1+z^2\varphi_2$

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