ARTICLE IN PRESS

Applied Mathematical Modelling xxx (2013) xxx–xxx

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Analytical solutions of refined plate theory for bending, buckling and vibration analyses of thick plates

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ARTICLE INFO

Article history:
Received 22 November 2011
Received in revised form 1 March 2013
Accepted 24 March 2013
Available online xxxx

Keywords:
Analytical solution
Bending
Buckling
Vibration
Refined plate theory

ABSTRACT

Analytical solutions for bending, buckling, and vibration analyses of thick rectangular plates with various boundary conditions are presented using two variable refined plate theory. The theory accounts for parabolic variation of transverse shear stress through the thickness of the plate without using shear correction factor. In addition, it contains only two unknowns and has strong similarities with the classical plate theory in many aspects such as equations of motion, boundary conditions, and stress resultant expressions. Equations of motion are derived from Hamilton's principle. Closed-form solutions of deflection, buckling load, and natural frequency are obtained for rectangular plates with two opposite edges simply supported and the other two edges having arbitrary boundary conditions. Comparison studies are presented to verify the validity of present solutions. It is found that the deflection, stress, buckling load, and natural frequency obtained by the present theory match well with those obtained by the first-order and third-order shear deformation theories.

1. Introduction

In order to solve the plate problems, two main steps must be taken: the choice of plate theory and the choice of solution method. Most commonly used plate theories can be classified into three main categories based on considering transverse shear deformation effects: (1) classical plate theory (CPT), (2) first-order shear deformation theory (FSDT), and (3) higher-order shear deformation theory (HSDT). The governing equation derived from the aforementioned theories can be solved using either numerical method (e.g., finite element method, differential quadrature method, mesh-free method, Ritz method, Galerkin method), or analytical method (e.g., power series method).

The CPT is the simplest one in which the normal to the mid-plane remains straight and normal to the middle surface during the deformation. The CPT has been applied to the bending, buckling, and vibration analyses of plates using analytical method [1–4] and numerical method [5–8]. Due to ignoring the transverse shear deformation effects, the CPT provides accurate results for thin plate. For thick plate, it underestimates deflections and overestimates buckling loads and natural frequencies. To overcome the deficiency of the CPT, the FSDT [9,10] was proposed. The FSDT accounts for the transverse shear effect but requires a shear correction factor to satisfy the free transverse shear stress conditions on the top and bottom surfaces of the plate. A large number of researchers have employed the FSDT to analyze the bending, buckling, and vibration responses of moderately thick plate using both analytical method [11–16] and numerical method [17–33]. Although the FSDT provides a sufficiently accurate description of response of thin to moderately thick plates, it

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0307-904X/$ - see front matter © 2013 Published by Elsevier Inc.
http://dx.doi.org/10.1016/j.apm.2013.03.038

is not convenient to use due to difficulty in determination of correct value of shear correction factor. To avoid the use of shear correction factor and obtain better prediction of response of thick plate, a number of HSDTs have been developed based on assumption of quadratic, cubic or higher-order variations of in-plane displacements through the thickness, notable among them are Lo et al. [34], Kant [35], Reddy [36], Bhimaraddi and Stevens [37], Touratier [38], Subramanian [39], Hanna and Leissa [40], and Guangyu [41]. Among them, the third-order shear deformation theory (TSDT) of Reddy [36] is widely used [42–46]. Although the HSDTs offer a slight improvement in accuracy compared to FSDT, their equations of motion are much more complicated than those of FSDT. Therefore, Shimpi [47] develop a two variable refined plate theory which is simple to use.

The Shimpi's theory is based on the assumption that the in-plane and transverse displacements consist of bending and shear components in which the bending components do not contribute toward shear forces and, likewise, the shear components do not contribute toward bending moments. The most interesting feature of this theory is that it accounts for a quadratic variation of the transverse shear strains across the thickness, and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. In addition, it has strong similarities with CPT in some aspects such as equations of motion, boundary conditions and stress resultant expressions. Because of its accuracy and simplicity, it was extended to other materials (e.g., orthotropic, laminated composite, functionally graded materials) with different problems by Shimpi and other investigators. For example, Shimpi and Patel [48] extended the RPT to the vibration of isotropic plates. The application of the RPT to orthotropic plates was carried out by Shimpi and Patel [49] for the bending and vibration problems and by Kim et al. [50] for the buckling problems. Thai and Kim [51–53] derived the Levy solutions of the RPT for the bending [51], buckling [52], and vibration [53] of orthotropic plates. The application of the RPT to laminated composite plates was done Kim et al. [54] for the bending and buckling problems and by Thai and Kim [55] for the vibration problems. Vo and Thai [56] adopted the RPT for buckling and vibration analyses of laminated beams. Recently, the RPT is extend to nanobeams [57], nanoplates [58,59], functionally graded sandwich plates [60], and functionally graded plates [61–63].

So far, no literature has been reported for Levy solutions of RPT for isotropic plates. Therefore, this paper aims to derive Levy solutions of RPT for bending, buckling and vibration analyses isotropic plates. Equations of motion are derived from Hamilton's principle. Analytical solutions of deflection, buckling load, and natural frequency are obtained for rectangular plates with two opposite edges simply supported and the other two edges having arbitrary boundary conditions. The accuracy of obtained solutions is verified by comparing the present results with three-dimensional elasticity solutions and those predicted by CPT, FSDT, and TSDT. Finally, some new results are presented to serve as references for comparison with further plate models.

2. Equations of motion

The two variable refined plate theory satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. The displacement field of this theory is as follow [47]

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) - z\frac{\partial w_b}{\partial x} + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x} \\ u_2(x,y,z,t) &= v(x,y,t) - z\frac{\partial w_b}{\partial y} + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial y} \\ u_3(x,y,z,t) &= w_b(x,y,t) + w_s(x,y,t), \end{aligned} \tag{1}$$

where (u_1, u_2, u_3) are the total displacement along the coordinates (x, y, z); (u, v) are the in-plane displacements on the middle plane along the coordinates (x, y); w_b and w_s are the bending and shear components of transverse displacement u_3, respectively; h is the plate thickness; and t is the time.

The linear strains can be obtained as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \kappa_x^b \\ \kappa_y^b \\ \kappa_{xy}^b \end{Bmatrix} + f\begin{Bmatrix} \kappa_x^s \\ \kappa_y^s \\ \kappa_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g\begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix}, \tag{2}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} \kappa_x^b \\ \kappa_y^b \\ \kappa_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_b}{\partial x^2} \\ -\frac{\partial^2 w_b}{\partial y^2} \\ -2\frac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \kappa_x^s \\ \kappa_y^s \\ \kappa_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_s}{\partial x^2} \\ -\frac{\partial^2 w_s}{\partial y^2} \\ -2\frac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w_s}{\partial y} \\ \frac{\partial w_s}{\partial x} \end{Bmatrix}, \tag{3}$$

$$f = -\frac{1}{4}z + \frac{5}{3}z\left(\frac{z}{h}\right)^2, \quad g = 1 - \frac{df}{dz} = \frac{5}{4} - 5\left(\frac{z}{h}\right)^2.$$

Please cite this article in press as: H.-T. Thai, D.-H. Choi, Analytical solutions of refined plate theory for bending, buckling and vibration analyses of thick plates, Appl. Math. Modell. (2013), http://dx.doi.org/10.1016/j.apm.2013.03.038