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The impact of stock-dependent demand on supply chain dynamics

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ABSTRACT

This paper presents an investigation on the dynamics of a supply chain system under stock-dependent demand. Considering the feature of piecewise linearity, a switched linear model composed of three subsystems is developed. Based on the switched model, some analytical stability results are derived. Simulation experiments are designed to verify the stability results and observe nonlinear dynamics. We show that stock-dependent demand not only leads to different stability results but also makes nonlinear dynamics more complicated. We also reveal that the nonlinear dynamics of the switched model, such as chaotic and periodic fluctuations of inventory and order, are essentially caused by switching frequently among subsystems due to uncertainties of inventory status. The results obtained in this paper help us understand the dynamic complexities of supply chain system and provide guidelines for selecting decision parameters to improve overall performance.

1. Introduction

Increasing competition in the market usually results in high fluctuations of inventory and demand in supply chain systems. It is commonly believed that a supply chain system can exhibit complicated dynamics, such as bullwhip effect [1], stability [2–4], and chaos [5–7]. These dynamic behaviors are closely related to system performance. For example, the bullwhip effect causes huge operating costs for upstream suppliers, while instability and chaos induce extra costs because of large variations of inventory and order [5]. Therefore, it is increasingly important for us to understand the dynamics before we can manage a supply chain system well.

The dynamic complexities in supply chain systems result from many factors: lead times, ordering rules, capacity constraints, interactions between firms, etc. For example, Lee et al. identified five major causes of the bullwhip effect: demand signal processing, non-zero lead times, order batching, supply shortages, and price variations [1]. Lead time reduction is found to be very beneficial in reducing inventory and demand variability and improving customer service level and responsiveness. In addition, ordering rule, another important contributory factor to complex dynamics, plays an important role in controlling inventory level and determining order quantities [8]. On the contrary, inventory also has a motive effect on the amount of demand. In other words, demand might depend upon the amount of the inventory displayed by retailers. It has been observed in supermarkets that demand is usually influenced by the amount of stock displayed on shelves [9]. There are also some literature showing this promotional effect of inventory from real data. For example, Koschat provided the evidence that the demand of US magazine industry can indeed vary with inventory [10]. Actually, inventory might increase sales for various reasons, e.g., increasing product visibility, kindling latent demand, and signaling a popular product [11].

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This paper studies the dynamics of a supply chain system composed of one external supplier and one retailer under one kind of stock-dependent demand function, in which demand is a piecewise linear function of inventory level with two demand parameters: one is used to specify the minimal demand while the other is used to represent the magnitude of the dependence of demand on inventory. In our model, the retailer uses an adaptive ordering rule. The ordering rule is a generalized order-up-to policy with two key decision parameters: one is used to specify the target of inventory level while the other is used to restore the retailer's inventory to a desired level. This ordering policy is selected as it has the capability and flexibility to improve supply chain dynamics by adjusting the values of decision parameters. The main objective of this paper is to explore how both demand and decision parameters affect dynamics and system performance.

The contribution of this paper is as follows:

(1) The dynamics of the supply chain system under stock-dependent demand, to our knowledge, have not been well addressed in the literature. Without considering the interaction between supply chain members and customers might make the obtained results unreliable or incomplete. In this paper, we show that stock-dependent demand essentially changes the dynamic properties in contrast to exogenous demand.
(2) The vast majority of literature on nonlinear dynamics of supply chain system has been limited to simulation methods. In this research, we represent the dynamic model as a switched linear system to capture the characteristics of piecewise linearity. Switched system theory might open the possibilities for studying nonlinearities in supply chain systems analytically. Although switched system theory has been extensively applied in the field of control engineering [12], there is only a little research using switched system theory to study supply chain systems [13].
(3) Our results reveal that stock-dependent demand not only leads to different stability results but also makes the nonlinear dynamics more complicated. Up to now, the causes and effects of nonlinear dynamics of supply chain systems are still not clear. In this paper, we attribute the causes of nonlinear dynamics to frequent switching among subsystems due to uncertainties of inventory state.

This paper is organized as follows. The next section provides a brief review on the related literature. Section 3 describes the supply chain system with stock-dependent demand and develops a switched linear model. Section 4 studies the stability of the switched linear system. In Section 5, simulation experiments are designed to explore the causes and effects of complex dynamics. This paper is concluded in Section 6.

2. Literature review

This paper is related to two streams of literature. The first stream deals with the problem of stock-dependent demand, while the second focuses on the dynamics of supply chain systems. In this section, we shall now review the literature on stock-dependent demand and supply chain dynamics.

2.1. Stock-dependent demand

The problem of stock-dependent demand has been studied by both empirical and theoretical papers. Empirical evidence of the dependence of demand on inventory of specific products has been provided in [14,15,10]. Instead, theoretical models are proposed for developing optimal ordering policies under different assumptions [16–20]. For example, Baker and Urban [21] established an economic order quantity model for power-form demand function. Balakrishnan et al. derived analytical results to gain insights on how to manage stock-dependent demand [11]. Wu et al. [22] proposed an optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Chang et al. [16] extended the model proposed in [22] by reconsidering the objective function and constraints. Min et al. [23] developed a lot-sizing model for deteriorating items with a current-stock-dependent demand and delay in payments. Stavrunlaki [24] investigated the joint effect of production substitution and demand stimulation. Differently, this paper focuses on the detailed dynamics of a supply chain with stock-dependent demand, namely, the fluctuations of inventory and order in the operating process.

2.2. Supply chain dynamics

Supply chain dynamics can be grouped into linear dynamics and nonlinear dynamics. The linear dynamics, such as stability, can be studied with control engineering methods, and the nonlinear dynamics, for example, chaos, have been investigated mostly by simulation experiments.

In the past, many control engineering techniques, such as transfer function, filter theory, and spectral analysis, are used to analyze stability and bullwhip effect of linear supply chain systems [4,25–27]. For example, Disney and Towill [4] applied the discrete transfer function model to investigate the stability of a vendor-managed inventory (VMI) model with exogenous demand. They illustrated the dynamic response for different replenishment parameters. Dejonckheere et al. [8] presented a methodology for measuring the bullwhip effect from a control engineering perspective. The methodology was based on a three-step procedure: deriving the transfer function, plotting the curve of frequency response, and calculating the noise