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Analysis of functionally graded plates using higher order shear deformation theory

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ABSTRACT

This work addresses a static analysis of functionally graded material (FGM) plates using higher order shear deformation theory. In the theory the transverse shear stresses are represented as quadratic through the thickness and hence it requires no shear correction factor. The material property gradient is assumed to vary in the thickness direction. Mori and Tanaka theory (1973) [1] is used to represent the material property of FGM plate at any point. The thermal gradient across the plate thickness is represented accurately by utilizing the thermal properties of the constituent materials. Results have been obtained by employing a C° continuous isoparametric Lagrangian finite element with seven degrees of freedom for each node. The convergence and comparison studies are presented and effects of the different material composition and the plate geometry (side-thickness, side–side) on deflection and temperature are investigated. Effect of skew angle on deflection and axial stress of the plate is also studied. Effects of material constant *n* on deflection and the temperature distribution are also discussed in detail.

1. Introduction

Functionally graded materials are a new class of composite materials where the material properties vary continuously to yield a predetermined composition profile. The functionally graded materials are microscopically heterogeneous and made from two isotropic materials such as metals and ceramics. The functionally graded material plates/panels have received considerable attention among the researchers in past two decades due to its ideal performance e.g., high heat resistance of ceramics on one side, and large mechanical strength and toughness of metal on the other side. Unlike, composite materials they have smooth and continuous variation across the adjoining layers. By spatially varying the microstructure, the material can be tailored for a particular application to yield optimal thermal and mechanical behavior. Due to the above mentioned reasons FGM have gained many applications in a wide variety of engineering fields which include nuclear structures, biomedical engineering, electrical engineering, aircraft engineering, nano FGM and has various industrial applications also. The following literature review provides a background for the present research.

Reddy [2] presented Navier's solution for the analysis of FGM rectangular plates and considered the effect of geometric non-linearity into account. In the study, finite element model is presented based on third order shear deformation theory. Analysis of skew rhombic plates using simple three noded element is performed by Sengupta [3]. Analysis of parallelogram shaped plates using a Mindlin nine noded heterosis elements is carried out by Butalia et al. [4]. The performance of the element was checked by adopting large skew angles under different boundary and loading conditions. Qian et al. [5] have used

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the meshless local Petrov–Galerkin method for the static analysis of thick rectangular functionally graded elastic plate. They concluded that the response of the composite plate lies between ceramic and metal plate when static and dynamic analysis has been performed. Static analysis of the functionally graded plate by collocation multiquadric radial basis functions is given by Ferreira et al. [6]. It is found that, if the Poisson's ratio has wide variation between ceramic and metal then the two homogenization schemes namely, Voigt rule of mixture and Mori–Tanaka method produces quite different results. Talha and Singh [7] performed the static analysis of functionally graded material plates by higher order shear deformation theory with a special modification in the transverse displacement. However, they have not considered any thermal effect in the analysis. Wu et al. [8] presented the static and dynamic responses of the functionally graded materials rectangular plate using quadratic extrapolation technique and finite double Chebyshev series for discretization. Analysis of symmetric and un-symmetric skew composite plates is carried out by Chakrabarti et al. [9] using new triangular finite element. Praveen and Reddy [10] analyzed nonlinear static and dynamic response of FGM plates subjected to mechanical and thermal load based on the Mindlin's first order shear deformation plate theory. Exact solutions for static and dynamic deformations of a FGM plate are given by Vel and Batra [11]. They utilized Mori–Tanka scheme and self consistent scheme to assess the material properties of the plate at different locations.

From the literature review it is observed that vey less work is carried out on analysis of FGM plates using higher order shear deformation theory and so far, there is no literature available for static analysis of FGM considering accurate temperature distribution based on the thermal properties of the constituent materials. Also, there is no result available in the literature for static analysis of FGM skew plates. Hence in the present paper, an attempt has been made to develop a finite element model based on higher order shear deformation theory that requires no shear correction. A C_o finite element with seven degrees of freedom per node is proposed to carry out the analysis. It is assumed that material properties of FGM are changed gradually along the thickness of the plate from a ceramic rich surface to a metal rich surface. Mori and Tanaka theory [1] is used to represent the material property of FGM at any point, which includes the Young's modulus (E), Poisson's ratio (γ), thermal conductivity (k), thermal expansion (α) and temperature (T). The effects of volume fraction (n) of the material constituents on the static response of the FGM plate with different combinations of the boundary conditions and plate geometry are investigated.

2. Homogenization of material properties

The FGM plate is made of two randomly distributed isotropic constituents (ceramic and metal), the macroscopic response of the composite is isotropic, and the composition of the composite varies only in the thickness direction. By power law distribution, the effective property of the functionally graded material plate at any height z can be expressed as

$$\begin{aligned} &P(z) = (P_c - P_m)V_c(z) + P_m \\ &\text{where } V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n, \quad 0 \leqslant n \leqslant \infty, \end{aligned} \tag{1}$$

where P_m and P_c denotes the material properties of the metal and ceramic, respectively. V_c is the volume fraction of the ceramic and n is the volume fraction index. Further, the volume fraction of the ceramic and metal phases are related by the equation $V_c + V_m = 1.0$.

According to the Mori–Tanaka scheme [1] the effective bulk modulus (B), and the effective shear modulus (G), modulus of Elasticity (E), Poisson's ratio (γ), effective heat conductivity coefficient (k), coefficient of thermal expansion (α) and temperature (T) at any point within the FGM plate are given by

$$\begin{aligned} &\frac{B-B_m}{B_c-B_m} = \frac{V_c}{1+(1-V_c)\frac{3(B_c-B_m)}{3B_m+4G_m}}, \\ &\frac{G-G_m}{G_c-G_m} = \frac{V_c}{1+(1-V_c)\frac{3(G_c-G_m)}{3G_m+f1}}, \quad \text{where} \quad f1 = \frac{G_m(9B_m+8G_m)}{6(B_m+2G_m)}, \\ &E = \frac{9GB}{3B+G}, \quad \gamma = \frac{3B-2G}{6B+2G}, \\ &\frac{k-k_m}{k_c-k_m} = \frac{V_c}{1+(1-V_c)\frac{(k_c-k_m)}{3k_m}}, \\ &\alpha = \alpha_m(1-V_c) + \alpha_c V_c + \frac{\alpha_c-\alpha_m}{\frac{1}{B_c}-\frac{1}{B_m}}\left(\frac{1}{B} - \frac{1-V_c}{B_m} - \frac{V_c}{B_c}\right), \\ &T(z) = T_m + (T_c - T_m)\eta(z,h), \end{aligned} \tag{2}$$

where

$$\eta(z,h) = \frac{1}{C}\left[\begin{array}{l} \left(\frac{2z+h}{2h}\right) - \frac{k_{cm}}{(n+1)k_m}\left(\frac{2z+h}{2h}\right)^{n+1} + \frac{k_{cm}^2}{(2n+1)k_m^2}\left(\frac{2z+h}{2h}\right)^{2n+1} - \frac{k_{cm}^3}{(3n+1)k_m^3}\left(\frac{2z+h}{2h}\right)^{3n+1} \\ + \frac{k_{cm}^4}{(4n+1)k_m^4}\left(\frac{2z+h}{2h}\right)^{4n+1} - \frac{k_{cm}^5}{(5n+1)k_m^5}\left(\frac{2z+h}{2h}\right)^{5n+1} \end{array}\right],$$

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