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Large-amplitude free vibrations of functionally graded beams by means of a finite element formulation

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ABSTRACT

The large-amplitude free vibration analysis of functionally graded beams is investigated by means of a finite element formulation. The Von-Karman type nonlinear strain–displacement relationships are employed where the ends of the beam are constrained to move axially. The effects of the transverse shear deformation and rotary inertia are included based upon the Timoshenko beam theory. The material properties are assumed to be graded in the thickness direction according to the power-law distribution. A statically exact beam element which devoid the shear locking effect with displacement fields based on the first order shear deformation theory is used to study the geometric nonlinear effects on the vibrational characteristics of functionally graded beams. The finite element method is employed to discretize the nonlinear governing equations, which are then solved by the direct numerical integration technique in order to obtain the nonlinear vibration frequencies of functionally graded beams with different boundary conditions. The influences of power-law exponent, vibration amplitude, beam geometrical parameters and end supports on the free vibration frequencies are studied. The present numerical results compare very well with the results available from the literature where possible. Some new results for the nonlinear natural frequencies are presented in both tabular and graphical forms which can be used for future references.

1. Introduction

The large-amplitude vibration analysis of beams has been studied by various researches using analytical and numerical techniques. The preliminary work dealing with the geometrically nonlinear vibrations of beams was performed by Woinowsky-Krieger [1]. He investigated the nonlinear vibrations of hinged-hinged beams with axially immovable ends using the elliptic integral solution. This problem was afterwards studied using perturbation and Ritz–Galerkin methods [2–4]. There can be found some newer analytical studies on the nonlinear vibrations of beam which seem to be useful [5–8]. The early publications conducted on the finite element (FE) vibration analysis of beams were presented by Mei [9–11]. After that tremendous efforts have been made on finding the FE solutions for this problem. Venkateswara Rao et al. [12] formulated the large-amplitude free vibrations of beams and plates by linearizing the quadratic terms in the strain–displacement relations. However, they discarded the effect of the ax-

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ial displacement. Raju et al. [13] included the effect of the axial displacement into the consideration and used the same linearizing approach as in [12].

In all the above FE formulations, the assumption of the simple harmonic motion (SHM) is used which results in the satisfaction of the equations of motion only at the instant of maximum amplitude. Kapania and Raciti [14] studied the nonlinear free vibrations of composite beams. In this formulation they reduced the dynamic FE matrix equation to a scalar equation by using the linear mode shapes obtained with the assumption of the SHM and finally solved this scalar equation by means of perturbation method. The main deficiency about their formulation was the unsatisfaction of the out-of-plane equilibrium equations. Sing et al. [15] investigated the problem of nonlinear oscillations of beams by improving the solution procedure of previous FE works. Using the linear mode shape obtained via SHM, they iteratively solved the dynamic FE matrix equation. This leads to the exact satisfaction of the equations corresponding to the axial and out-of-plane directions. The converged eigenvector was then used in order to reduce the dynamic FE matrix equation to a scalar nonlinear Duffing-type one which finally solved using the direct numerical integration (DNI) technique. Gupta et al. [16] presented a relatively simple FE formulation and gave the natural nonlinear frequencies of Euler–Bernoulli beams with end supports of any type. Their FE formulation begins with the assumption of the SHM and is consequently corrected by implementing the harmonic balance method (HBM). Afterwards, they continued their work and studied the same problem based upon the Timoshenko beam theory [17]. However, most of the investigations conducted on the nonlinear vibration and buckling analysis of beams are concerned with isotropic and laminated composite beams and fewer studies deal with beams made up of functionally graded (FG) materials.

Xiang et al. [18] studied the free and forced vibration of FG Timoshenko beams of variable thickness under heat conduction using the differential quadrature method. Sina et al. [19] developed a new beam theory within the framework of the first order shear deformation theory to analyze free vibration of FG beams. In their study, the equations of motion are derived using Hamilton's principles which are solved then using an analytical approach. Simsek [20] investigated the large-amplitude vibrations of FG beams under the action of a moving load based on the Timoshenko beam theory. The equations of motion are derived using Lagrange's equation and finally solved by means of Newmark-β method in conjunction with the direct iteration method. Fallah and Aghdam [21] implemented He's variational method [22] to investigate the nonlinear characteristics of FG beams resting on nonlinear elastic foundations where the Euler–Bernoulli assumptions together with Von-Karman's strain–displacement relations are employed for deriving the governing equations of motion. The number of publications dealing with the nonlinear finite element vibration analysis of FG beam is scarce. To the best of author's knowledge, the most recent work in this field is limited to the linear vibration analysis of FG beams using FE method which was performed by Alshorbagy et al. [23]. In their work, the equations of motion are derived using the Euler–Bernoulli beam theory and virtual work principle. The material properties of beams assumed to be varying through the thickness or longitudinal directions according to the power-law volume fraction function.

Herein, a study is carried out on the large-amplitude oscillations of FG beams using the Timoshenko beam theory which includes the effects of rotary inertia and transverse shear deformation in the dynamic analysis. Statically exact beam element with displacement fields based on the first order shear deformation theory is used for the finite element discretization of the problem. This means that the shape functions are achieved based on the exact satisfaction of the static part of the governing differential equations and thus give exact stiffness matrices and a more accurate mass matrix (For further studies the reader is referred to [24,25]). Therefore, the application of this beam element whose shape functions are functions of length, cross sectional and material properties of the beam, gives more accurate numerical results compared to the other beam element whose shape functions are linear. Agrawal et al. [26] examined the application of this kind of beam element for the large deformation analysis of anisotropic and inhomogeneous beams. Their results illustrated that the exact shape function based element has a faster rate of convergence than the beam elements formed using linear shape functions. The FG beam is considered with different boundary conditions and both ends are constrained to move axially, resulting in the Von-Karman type strain–displacement relation. The material properties are assumed to be graded in the thickness direction according to the power-law distribution. The linear mode corresponding to the fundamental frequency is utilized to reduce the dynamic matrix equation to a scalar one. Finally, the direct numerical integration method [15,27] is applied for solving this scalar equation. Finally, the influences of different boundary conditions, power exponent index and beam's length to the thickness ratio on the nonlinear vibration frequencies are examined. A comparison is made also with the results obtained via the Euler–Bernoulli beam theory which clarifies the overestimation of the frequencies by the latter one.

2. Material properties of FGM material

A FG beam composed of steel and alumina of length L, width b and thickness h is shown in Fig. 1. It is assumed that the material properties of the beam such as, Young's modulus E, mass density ρ and Poisson's ratio ν vary continuously along the thickness direction across the thickness based on the following relations