Applied Mathematical Modelling 37 (2013) 8505–8517

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A new more consistent Reynolds model for piezoviscous hydrodynamic lubrication problems in line contact devices ☆

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ARTICLE INFO

Article history:
Received 4 October 2012
Received in revised form 11 March 2013
Accepted 24 March 2013
Available online 11 April 2013

Keywords:
Hydrodynamic lubrication
Piezoviscous regime
Reynolds equation
Cavitation phenomenon
Free boundary

ABSTRACT

Hydrodynamic lubrication problems in piezoviscous regime are usually modeled by the classical Reynolds equation combined with a suitable law for the pressure dependence of viscosity. For the case of pressure–viscosity dependence in the Stokes equation, a new Reynolds equation in the thin film limit has been proposed by Rajagopal and Szeri. However, these authors consider some additional simplifications. In the present work, avoiding these simplifications and starting from a Stokes equation with pressure dependence of viscosity through Barus law, a new Reynolds model for line contact lubrication problems is deduced, in which the cavitation phenomenon is also taken into account. Thus, the new complete model consists of a nonlinear free boundary problem associated to the proposed new Reynolds equation.

Moreover, the classical model, the one proposed by Rajagopal and Szeri and the here proposed one are simulated through the development of some numerical algorithms involving finite elements method, projected relaxation techniques, duality type numerical strategies and fixed point iteration techniques. Finally, several numerical tests are performed to carry out a comparative analysis among the different models.

1. Introduction

In thin film lubrication problems the assumption that the viscosity of the lubricant remains constant, and therefore independent of pressure, results to be reasonable in low pressure regimes. However, when the lubricant is subjected to high pressures a strong dependence of viscosity on pressure arises, thus leading to the so called piezoviscous lubrication regimes. In the mechanical and mathematical literature concerning the models for piezoviscous hydrodynamic thin film lubrication problems, different classical devices have been considered, such as journal-bearings, rolling-bearings or rolling-ball-bearings (see [1], for example). In all of these situations, the behavior of the lubricant pressure in the thin film setting has been classically modeled by the Reynolds equation, in which the pressure dependence of viscosity is usually introduced *a posteriori* by some expression. In this procedure, the thin film limit from Stokes equation to Reynolds one is obtained regardless of the pressure–viscosity dependence, as in the case of constant viscosity (isoviscous regime). We note that in the constant viscosity case, the heuristic statement of incompressible Reynolds equation from Stokes model is obtained in [2] while the more rigorous one based on asymptotic developments is proved in [3].

☆ This paper has been partially funded by MEC (Project MTM2010-21135-C02).
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0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2013.03.072

In the piezoviscous case, one of the most classical expressions relating pressure and viscosity is given by the Barus law

$$\mu = \mu_0 e^{\alpha p}, \tag{1}$$

where μ, μ_0, p and α denote the viscosity, the zero pressure viscosity, the pressure and the piezoviscosity coefficient, respectively. Classically in the lubrication literature, the expression (1) is plugged into Reynolds equation to model piezoviscous regimes. The resulting nonlinear Reynolds equation is then used inside more complex models that additionally consider presence of cavitation or elastohydrodynamic phenomena. In these more complex settings, this way of including piezoviscous regimes has given rise to several mathematical analysis results that state the existence and the uniqueness of solution, as well as to the design of suitable numerical methods to approximate the corresponding solutions, for which there are no analytical expressions (see, for example, [4–8]).

Barus law has been also used to model pressure dependence of viscosity in the original Stokes equation in [9] arguing also that Reynolds equation is only valid when the shear stress is much smaller than the reciprocal of the pressure–viscosity coefficient, while later on in [10] a corrected Reynolds equation is obtained from Navier–Stokes ones with pressure dependence of viscosity and in [11] a simpler derivation is obtained.

More recently, by assuming that the viscosity depends on pressure in Stokes equation according to Barus law, in [12] a more careful derivation of the limit Reynolds equation is carried out. The authors find additional terms to those ones appearing in the classical Reynolds equations used for elastohydrodynamic computations. Furthermore, they evaluate the consequences of these additional terms in the hydrodynamic regimes leading to high enough pressure values. This derivation starts from the assumptions that stress tensor in the incompressible fluid includes the possibility that constraint forces could influence the work and is linear in the symmetric part of the velocity gradient tensor. Thus, unlike the Navier–Stokes equation for incompressible fluids which involves an explicit relation between both tensors and a constant viscosity, in this departure model an implicit relation holds and pressure dependence of viscosity is considered. More precisely, the departure model in the case of a steady two dimensional plane flow is given by the following equations:

$$-\frac{\partial p}{\partial x} + \mu(p)\Delta u + 2\mu'(p)\frac{\partial u}{\partial x}\frac{\partial p}{\partial x} + \mu'(p)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\frac{\partial p}{\partial y} = \rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] \tag{2}$$

$$-\frac{\partial p}{\partial y} + \mu(p)\Delta v + \mu'(p)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\frac{\partial p}{\partial x} + 2\mu'(p)\frac{\partial v}{\partial y}\frac{\partial p}{\partial y} = \rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$

where x and y denote the spatial coordinates, (u, v) represents the velocity field and ρ denotes the fluid density. Note that the viscosity depends on pressure. After some simplifying assumptions detailed in [12], including that $\partial p/\partial y = 0$, Eqs. (2) and (3) are reduced to

$$\frac{dp}{dx} = \mu\frac{\partial^2 u}{\partial y^2} + 2\frac{d\mu}{dp}\frac{\partial u}{\partial x}\frac{dp}{dx}. \tag{5}$$

Next, by integrating (5) twice across the film and taking into account the boundary conditions for the horizontal velocity we can get an expression for $u(x, y)$. Finally, by replacing this expression in (4) and integrating again across the film for a vertical velocity v vanishing at both surfaces, the following modified Reynolds equation is obtained:

$$\frac{d}{dx}\left[\left(\frac{h^3}{\mu} - 12\alpha\int_0^h y(h-y)\frac{\partial u}{\partial x}dy\right)\frac{dp}{dx}\right] = 6s\frac{dh}{dx}, \tag{6}$$

jointly with the Barus law (1). Here $h, (u, v)$ and $(s, 0)$ denote the gap between surfaces, the velocity field in the thin film and the given velocity of the lower sliding surface, assuming that the upper surface is fixed. Furthermore, by using the simplification

$$\frac{\partial u}{\partial x} \approx \frac{du_{av}}{dx}, \tag{7}$$

where u_{av} is an average velocity, the flow rate

$$Q = h(x)u_{av}(x) \tag{8}$$

is introduced in [12] to deduce the following modified Reynolds equation

$$\frac{d}{dx}\left[\left(\frac{h^3}{\mu} + \alpha Q\frac{dh^2}{dx}\right)\frac{dp}{dx}\right] = 6s\frac{dh}{dx}. \tag{9}$$

Next, for the case of the hydrodynamic and elastohydrodynamic lubrication of a long cylinder rolling on a plane, in [12] the effect of the extra term appearing in the modified equation with respect to the classical one is numerically investigated. The chosen device parameters imply that high pressures appear, so that their computed values and those ones of the corresponding viscosities allow to illustrate the existence of differences between both models, mainly consisting in higher values for the case of modified equation.