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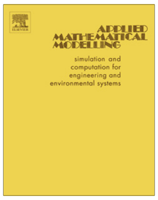

Capturing coalescence and break-up processes in vertical gas–liquid flows: Assessment of population balance methods

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ABSTRACT

Gas–liquid flows are commonly encountered in industrial flow systems. Numerical studies have been performed to assess the performances of different population balance approaches – direct quadrature method of moments (DQMOMs), average bubble number density (ABND) model and homogeneous MUlti-SIze-Group (MUSIG) model – in tracking the changes of gas void fraction and bubble size distribution under complex flow conditions and to validate the model predictions against experimental measurements from medium- and large-sized vertical pipes. Subject to different gas injection method and flow conditions, bubble size evolution exhibited a coalescence dominant trend in the medium-sized pipe; while bubble break-up was found to be dominant in large-sized pipe. The two experiments were therefore strategically selected for carrying out a thorough examination of existing population balance models in capturing the complicated behaviour of bubble coalescence and break-up. In general, predictions of all the different population balance approaches were in reasonable agreement with experimental data. More importantly, encouraging results have been obtained in adequately capturing the dynamical changes of bubbles size due to bubble interactions and transition from *wall peak* to *core peak* gas void fraction profiles. As a compromise between numerical accuracy and computational time, DQMOM has performed rather well in capturing the essential two-phase flow structures within the medium- and large-sized vertical pipes when compared to those of ABND and homogeneous MUSIG models. From a practical perspective, the ABND model may still be considered as a more viable approach for industrial applications of gas–liquid flow systems.

1. Introduction

Two-phase gas–liquid flows exist in many industries: chemical, civil, nuclear, mineral, energy, food, pharmaceutical and metallurgy. Because of the complex two-phase flow structures that are usually found in these technological systems and since such flow structures can evolve dynamically and transit to different flow regimes, the phenomenological understanding of bubble size or interfacial area and its dispersion behavior is of paramount importance. Relevant experimental observations have revealed clear tendencies of the bubbles within the bulk liquid flow to undergo significant coalescence and break-up as well as deformation, evaporation and condensation within the particular system of interest subject to local flow

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Nomenclature

a	coalescence rate
$a(M_i, M_j)$	coalescence rate in terms of mass
A	coefficient matrix
$b(M_i, M_j)$	break-up rate in terms of mass
a_{if}	interfacial area concentration
B_B, B_C	mass birth rate due to break-up and coalescence for MUSIG
B_k^B, B_k^C	mass birth rate due to break-up and coalescence for DQMOM
C	break-up model constant
C_D	drag coefficient
C_f	coefficient of surface area
C_L	lift coefficient
C_{RC1}, C_{RC2}, C_{RC3}	coalescence coefficients for ABND
C_{TI1}, C_{TI2}	break-up coefficients for ABND
C_{w1}, C_{w2}	wall lubrication coefficients
C_{TD}	dispersion coefficient
D_H	maximum bubble horizontal dimension
d_{ij}	equivalent diameter
D_s	Sauter mean bubble diameter
D_B, D_C	mass birth rate due to break-up and coalescence for MUSIG
D_k^B, D_k^C	mass death rate due to break-up and coalescence for DQMOM
Eo	Eötvos number
Eo_d	modified Eötvos number
f	size fraction
f_{BV}	break-up volume fraction
F^{lg}	total interfacial force
F_B	break-up calibration factor
F_C	coalescence calibration factor
F^{lg}_{drag}	drag force
F^{lg}_{lift}	lift force
$F^{lg}_{lubrication}$	wall lubrication force
$F^{lg}_{dispersion}$	turbulent dispersion force
h_o	initial film thickness
h_f	critical film thickness
j	superficial velocity
k	turbulent kinetic energy
m^k	moments of particle (bubble) size distribution
M	mass scale of gas phase (bubble)
n_w	outward vector normal to the wall
n	average bubble number density or weight
P	pressure
r	break-up rate
Re_b	bubble Reynolds number
R	net rate of source and sink terms for ABND
S_i	mass transfer rate due to coalescence and break-up
S_k	moment source term
Sc_b	turbulent bubble Schmidt number
t	physical time
t_{ij}	time for two bubbles to coalesce
$\mathbf{u}$	velocity vector
u_t	turbulent velocity
v	volume of bubble
We	Webber number
We_{cr}	critical Webber number
y_w	distance from the wall boundary
Greek symbols	
α	void fraction
α_{max}	maximum allowable void fraction
β	break-up kernel constant

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