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An improved method for ranking alternatives in multiple criteria decision analysis

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ABSTRACT

Scoring rules are an important disputable subject in data envelopment analysis (DEA). Various organizations use voting systems whose main object is to rank alternatives. In these methods, the ranks of alternatives are obtained by their associated weights. The method for determining the ranks of alternatives by their weights is an important issue. This problem has been the subject at hand of some authors. We suggest a three-stage method for the ranking of alternatives. In the first stage, the rank position of each alternative is computed based on the best and worst weights in the optimistic and pessimistic cases, respectively. The vector of weights obtained in the first stage is not a singleton. Hence, to deal with this problem, a secondary goal is used in the second stage. In the third stage of our method, the ranks of the alternatives approach the optimistic or pessimistic case. It is mentionable that the model proposed in the third stage is a multi-criteria decision making (MCDM) model and there are several methods for solving it; we use the weighted sum method in this paper. The model is solved by mixed integer programming. Also, we obtain an interval for the rank of each alternative. We present two models on the basis of the average of ranks in the optimistic and pessimistic cases. The aim of these models is to compute the rank by common weights.

1. Introduction

In preferential voting systems, each voter selects k candidates from among n alternatives ($k \leqslant n$) and ranks them from the most to the least preferred. Each candidate may receive some votes in different ranking places. The total score of each candidate is the weighted sum of the votes he or she receives in different places. The value 1 is assigned to the most important alternative and n to the least important. Cook and Kress [1] used DEA to determine the most appropriate weights for each candidate. Their method is proved to be effective in voting systems. Green et al. [2] modified Cook and Kress's approach. Noguchi et al. [3] used the cross-efficiency assessment to obtain the best candidate and gave a strong ordering constraint condition on weights. Wang and Chin [4] distinguished efficient candidates by considering their least relative total scores. But the least relative total scores and the most relative total scores are not measured within the same range in [4]. Obata and Ishii [5] proposed non-DEA efficient candidates and discriminated between the DEA efficient candidates with normalized weights. Their method was subsequently extended to rank non-DEA efficient candidates in [6]. Wang et al. [7] proposed three new models for preference voting and aggregation. Wanga et al. [8] suggested a method for ranking efficient alternatives by comparing the most and least relative total scores of each efficient alternative. Hashimoto [9] introduced an AR/

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exclusion model based upon the contexts of super-efficiency presented in Andersen and Petersen [10]. Contreras [11] proposed a new model inspired by DEA methodology. He obtained the ranks of alternatives in two stages. In the first stage, he applied a DEA model to characterize the ranks of alternatives in their best condition. In order to obtain a group rank, he solved a multi-objective linear programming (MOLP) model in the second stage.

The cross-efficiency approach is a DEA tool that can be used to obtain best-performing DMUs and to determine the ranks of DMUs [12]. The main aim of cross-efficiency is to apply DEA for peer evaluation instead of self evaluation. Note that cross evaluation gives a unique ordering of the DMUs and can be used in many applications. Preference voting [2] is one of the situations where cross evaluation can be utilized. It should be noted that the non-uniqueness of the optimal weights in DEA models makes cross evaluation inapplicable. Sexton et al. [12] and Doyle and Green [13] applied secondary goals to resolve the non-uniqueness problem. They introduced two models, the aggressive and the benevolent models. The aggressive model obtains weights subject to minimizing the cross-efficiency of those other units, whereas the benevolent model shows the optimal weights based on maximizing the cross-efficiency of other units. As extension of Doyle and Green [13] model was introduced by Lianga et al. [14]. They presented different secondary objective functions that include an efficiency evaluation criterion. Wang and Chin [15] utilized a neutral DEA model for cross evaluation to avoid the difficulty in choosing between the two different formulations; a model that recognized one set of input and output weights for every DMU without being aggressive or benevolent to the others. Also, they employed the neutral DEA model for obtaining a common set of weights for DMUs.

In this paper, we introduce a three-stage method for the ranking of voting systems. In the first stage, the rank of each alternative is determined by its best weight. Thus, all alternatives are ranked by the best weight of the alternative under evaluation. In fact, each alternative is evaluated not only with its optimal weights but also with the remaining alternatives' ones. Therefore, the vector of weights derived is not a singleton. The score value is unique but not necessarily the vector that induces this value. So, the evaluation of the remaining alternatives can vary depending on which vector is selected. To resolve this problem, in the second stage, we suggest a secondary goal that limits the vector of weights. The ideal rank for each alternative is obtained from the best weights. For this purpose, we compute the average value of all ranks for each alternative and present the average rank, so that the alternative with a lower average has a better rank. In the third stage, the alternatives are ranked by common weights, as the ranks obtained have the minimum distance from the average rank. It must be mentioned that the ranks of alternatives are integer-valued and distinct but the proposed model has alternative ranks. Also, each alternative is evaluated with its best and worst weights. Thus, we can obtain a lower rank and an upper rank for each alternative. This means that we obtain an interval for the rank of each alternative. We present two models on the basis of the average of the ranks in the optimistic and pessimistic cases. The aim of these models is to compute the rank by common weights. In the third stage, the ranks of alternatives approach the optimistic or pessimistic case. In numerous cases, we would like the rank obtained for each alternative to have the minimum distance from the optimistic rank and the maximum distance from the pessimistic rank. It should be stated that the model proposed in the third stage is an MCDM model and there are several methods for solving it. We use the weighted sum method for this purpose. The model is solved by mixed integer programming.

We proceed as follows: In Section 2, a two-stage method for the ranking of voting systems is provided. Section 3 gives an improved method for ranking voting systems. The optimistic and pessimistic cases of group rank are presented in Section 4. Section 5 contains a numerical example, and Section 6 draws the conclusions.

2. A two-stage method for ranking voting systems

Suppose that we have n alternatives $x_j : j = 1,\ldots,n$ with $k(k \leqslant n)$ places. We use a voting system for ranking the alternatives. The rank vector of alternatives is from 1 to n. The value 1 is assigned to the most important alternative and n to the least important.

Contreras [11] introduced the two-stage model for the ranking of alternatives. In the first stage, the rank of each alternative is evaluated separately. The aim, here, is to minimize the rank position to describe what the best situation for an alternative is. To determine the best rank position of o-th alternative, r_o^o, Contreras [11] solved the following model in the first stage:

$$
\begin{aligned}
r_o^{o*} = \min \quad & r_o^o \\
\text{s.t.} \quad & (a)\ \sum_{j=1}^{k} w_j^o v_{ij} - \sum_{j=1}^{k} w_j^o v_{hj} + \delta_{ih}^o M \geqslant 0, \quad i = 1,\ldots,n,\ i \neq h, \\
& (b)\ \delta_{ih}^o + \delta_{hi}^o = 1, \quad i = 1,\ldots,n,\ i \neq h, \\
& (c)\ \delta_{ih}^o + \delta_{hk}^o + \delta_{ki}^o > 1, \quad i = 1,\ldots,n,\ i \neq h \neq k, \\
& (d)\ r_i^o = 1 + \sum_{h \neq i} \delta_{ih}^o, \quad i = 1,\ldots,n, \\
& (e)\ w^o \in \phi, \\
& (f)\ \delta_{ih}^o \in \{0,1\}, \quad i = 1,\ldots,n,\ i \neq h,
\end{aligned}
\tag{1}
$$