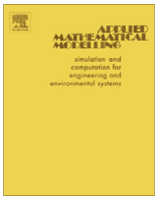
Applied Mathematical Modelling 37 (2013) 62–71

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm



A robust model for a leader–follower competitive facility location problem in a discrete space

Milad Gorji Ashtiani, Ahmad Makui *, Reza Ramezanian

Department of Industrial Engineering, Iran University of Science and Technology, Narmak, Tehran, Iran

ARTICLE INFO

Article history:
Received 30 December 2010
Received in revised form 21 November 2011
Accepted 4 December 2011
Available online 22 December 2011

Keywords:
Competitive location
Leader–follower
Robust Optimization
Potential location

ABSTRACT

This paper aims at determining the optimal locations for the leader's new facilities under the condition that the number of the follower's new facilities is unknown for the leader. The leader and the follower have some facilities in advance. The first competitor, the leader, opens p new facilities in order to increase her own market share. On the other hand, she knows that her competitor, the follower, will react to her action and locate his new facilities as well. The number of the follower's new facilities is unknown for the leader but it is assumed that the leader knows the probability of opening different numbers of the follower's new facilities. The leader aims at maximizing her own market share after the follower's new facilities entry. The follower's objective is also to maximize his own market share. Since the number of the follower's new facilities is unknown for leader, "Robust Optimization" is used for maximizing the leader's market share and making the obtained results "robust" in various scenarios in terms of different numbers of the follower's new facilities. The optimal locations for new facilities of both the leader and the follower are chosen among pre-determined potential locations. It is assumed that the demand is inelastic. The customers probabilistically meet their demands from all different facilities and the demand level which is met by each facility is computed by Huff rule. The computational experiments have been applied to evaluate the efficiency of the proposed model.

1. Introduction

Many factors should be considered for locating new facilities. One of the most important factors is existing/not existing competitors in the market that offer the same goods or services. When there is no competitor in the market, the facility which is going to be located will have monopoly condition. The vast part of location theory is to model a location problem with respect to the monopoly assumption. Practically, the aforementioned assumption and the respective models seldom become true and applicable in reality because a company rarely acts as the only player in the market and competition among different players seems to be more realistic. A review on this type of location problems can be seen in different papers [1,2].

"A location model is said to be about competitive facilities when it explicitly incorporates the fact that other facilities are already/will be present in the market and that the new facility/facilities will have to compete with them for its/their market share" [2]. Researches on competitive facility location models are originated by Hotelling [3]. He considered the competitive facility location problem under the conditions that customers are uniformly distributed on a line segment and each of competitors can locate her/his own facility at any locations in this space. All customers use the closest facility in Hotelling model.

---

* Corresponding author.
E-mail address: amakui@iust.ac.ir (A. Makui).

0307-904X/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.apm.2011.12.013

Huff [4] defined the attractive function of facility for customers by considering not only the distance but also the quality of facility. In fact, Huff formulated a model for capturing market share and considered the probability of customer patronizing behavior towards a facility. The aforementioned probability is proportional to the quality of the facility and inversely proportional to the squared distance between a facility and a customer.

Competitive facility location is categorized into three categories of (1) static competition, (2) competition with foresight and (3) dynamic competition. In this paper, the second category, i.e. competition with foresight will be considered. Competitive facility location models vary in the ingredients which form the model. For instance, the location space may be continuous, network or discrete. Demand is usually supposed to be concentrated in a discrete set of points, called demand points and it can be either inelastic or elastic depending on whether the goods or services are essential or inessential.

Competition with foresight has attracted the attention of many researchers who investigate the competitive location models. According to the Stackleberg's economic model in 1934, the aim of the leader–follower location problem is to find an optimal strategy for these two competitors who make decision sequentially. The leader–follower problem is called a Stackleberg game. This type of problems was introduced by Hakimi [5] for the first time. He used the expression "medianoid" for the follower's problem and "centroid" for the leader's problem. In fact, a $(r|X_p)$-*medianoid* problem is the one in which the follower locates r new facilities in order to maximize his objective function while the leader did the same action before. In other words, the leader is locating p facilities at a set of points X_p. In a $(r|p)$-*centroid* problem, the leader is following to find the optimal location of p new facilities and considering this fact that the follower will respond to the leader's action with locating r new facilities. Here, the maximization of leader's objective function is equivalent to the minimum of follower's objective functions which are maximized for their own problems. In general, when the demand is inelastic, the leader's problem will be a $(r|p)$-*centroid* problem. Hakimi has proved that the leader–follower problems in $(r|X_p)$-*medianoid* and $(1|p)$-*centroid* cases are NP-hard.

A few researches have been conducted about the leader's problem. Eiselt and Laporte [1] have reviewed all the researches that were undertaken about the leader–follower competitive location models till 1996. Various models in the network space have been studied by Hakimi [6,7]. Hakimi [6] solved the leader–follower problems in a network space with the aid of six scenarios which are the combination of elastic and inelastic demands and also three different rules of the customer behavior. Benati and Laporte [8] used heuristic methods for solving $(r|p)$-*centroid* problems. The leader's problem with deterministic behavior of customers in continuous space is discussed in different papers [9,10]. Drezner [11] solved the leader–follower problem for Hotelling model and Euclidean distance through geometry-based approach. Ghosh & Craig [9] have solved the similar problem to Drezner's one by making all variables discrete and also defining a set of predetermined potential locations for the leader–follower problems. They used integer programming for modeling the respective problem and their solution is only limited to relatively small scale problems. Two heuristics are proposed for the leader–follower problems in the continuous space and on the basis of Hotelling's proximity rule and Euclidean distance for locating new facilities [10]. On the basis of Huff rule [4], Drezner and Drezner [12] have proposed three heuristics about probabilistic behavior of customers in the continuous space for solving the leader–follower problems. In a research which was done by Sáiz et al. [13], the leader–follower problems for the single facility and on the basis of Huff rule are solved by branch and bound method. Redondo et al. [14] solved this problem with four heuristics and in a condition that the qualities of the new facilities are also considered as decision variables for the problem. Shiode et al. [15] have solved the leader–follower problems for the single facility in both linear and planar markets but it is assumed that the demand is inelastic and the customer behavior is deterministic and Hotelling-type. The distances in their study are considered as rectangular.

Alekseeva et al. [16] and Kononov et al. [17] have proposed a model for the leader–follower problems in a discrete space considering the rule of closest facility to the customers and maximizing the leader's and the follower's profit. Alekseeva et al. [16] has solved the mentioned model by a hybrid memetic algorithm. For the respective model, Alekseeva et al. [16] used function approximation for determining a set of upper and lower bounds. Alekseeva et al. [18] have developed his prior research [16] with offering a better solution. Plastria & Vanhaverbeke [19] have offered a model for the leader–follower problems in a discrete space. In their model, the follower locates a new facility while the leader does the same action with respect to her budget. In the aforementioned study, the customer behavior is considered deterministic and on the basis of the least distance and also three different strategies for the leader have been investigated. On the basis of probabilistic behavior of the customers and Huff rule, Gorji, Makui & Ramezanian [20] have solved the leader–follower model in a discrete space.

After reviewing the literature on this topic, this conclusion can be drawn that all previous papers have modeled the leader's problem with respect to the assumption of definite number of follower's new facilities but this assumption is relatively far from reality. The authors in this paper have eliminated the assumption of the definite number of the follower's new facilities and the leader's problem has been solved in a condition that the leader does not know that after locating her new facilities, how many facilities are going to be opened by the follower. Therefore, "Robust Optimization" is used here to remove this problem.

Robust Optimization (RO) is a branch of optimization theory. This method is used for problems in which an optimal robust solution against the uncertainty or variability in parametric value of the problem is sought. For the first time, Mulvey et al. [21] have posed the concept of this method in operation research in 1995. They offered an approach for optimization of objective function in a problem whose data are scenario-based. In this approach, they have used a penalty function in their non-linear objective function which is the expected values of different scenarios. After Mulvey's research, Ben-Tal and Nemirovski have developed the RO theory for linear quadratic and conic problems [22–26]. In fact, these authors utilized the continuous region for modeling uncertain parameters. Mulvey's RO approach is used in this paper.