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Aircraft proximity termination conditions in the planar turn centric modes

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ABSTRACT

Closed-form analytic solutions for proximity management strategies are of great importance as a design benchmark when validating both automated systems and procedures associated with the design of air traffic rules. Merz (1973) first presented a solution for a set of optimal strategies for resolving co-planar co-operative encounters between two aircraft (or ships) with identical linear and rotational speeds. This paper extends the solution domain for turning aircraft beyond that of identical aircraft by presenting a rigorous analysis of the problem through a generalised optimisation approach. This analysis provides a dependable method for determining the location of the point of closest approach. This is achieved by using a vector form of Fermat's equation for stationary points. A characteristic of this solution is the identification of a fixed reference point lying on the vector between the aircraft turn centres or on one of its extensions. This point is then used to determine where the location of the minima in the relative range between the aircraft will occur. Bounds for the domain of the solution are constructed in terms of the rotational angles of the aircraft on their turn circles. Four distinct topologies are required to characterise the types of minima that can occur. The methodology has applications in an operational context permitting a more detailed and precise specification of proximity management functions when developing algorithms for aircraft avionics and air traffic management systems.

1. Introduction

Systems that provide aircraft proximity management functions are currently under intensive review. This is due to the expected increase in traffic flow rates from three major areas: public demand for air travel, the introduction of uninhabited aerial vehicle operations (without a human pilot) and from the introduction of personalised jets. Concomitant with these increases is an expected increase in the frequency of aircraft proximity incidents. Close proximity situations can occur, for example, in the missed approach, in the circuit area, and for operations in uncontrolled airspace where aircraft routinely fly in close proximity and where air traffic management (ATM) services may be unavailable. These situations require dependable proximity management at physical limits well below the more commonly understood ATM separation standards used in the present controlled airspace and well below those presently proposed for situations such as Free Flight airspace. The assurances that minimum miss-distances as well as available response times will be preserved during close proximity situations are important objectives for engineering design (Refs. [1,2]) and for the conduct of flight operations (Refs. [3,4]).

The need for this research also arises within the airspace planning groups. Programmes such as NextGen in the USA and SESAR in Europe will require at least some re-design and validation of both conflict resolution and proximity management functions. Further, aircraft guidance that enhances the tracking and the prediction of flight paths is being investigated under new avionics architectures such as: "Pathway-in-the-Sky", "Tunnel-in-the-Sky", "Highway-in-the-Sky" (Refs. [5–15]) and "Cockpit Display for Traffic Information (CDTI)" initiatives (Refs. [16–20]).

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Nomenclature

a	turn radius of aircraft-*A*
b	turn radius of aircraft-*B*
$r = \frac{a}{b}$	ratio of turn radii (a non-dimensional quantity)
R	relative range from aircraft-*A* to aircraft-*B*
t	time variable
ζ	rotational angle of aircraft-*A*
γ	rotational angle of aircraft-*B*
ζ_0	initial angle of aircraft-*A*
γ_0	initial angle of aircraft-*B*
ζ^*	optimal rotational angle of aircraft-*A*
γ^*	optimal rotational angle of aircraft-*B*
$\omega_A, \underline{\omega_A}$	rotational speed (velocity) of aircraft-*A*
$\omega_B, \underline{\omega_B}$	rotational speed (velocity) of aircraft-*B*
$\omega = \frac{\omega_A}{\omega_B}$	ratio of rotational speeds (a non-dimensional quantity)
$Arc\sin(\bullet)$	principal value of the inverse sine
$Arc\tan(\bullet)$	principal value of the inverse tangent
$\mathbf{C}_A$	turning circle of aircraft-*A*
$\mathbf{C}_B$	turning circle of aircraft-*B*
$\underline{C}$	turn centre vector of turn circles from aircraft-*A* to aircraft-*B*
$C = \lVert\underline{C}\rVert$	turn centre distance
$\lVert\bullet\rVert$	the L_2 or Euclidean Norm of a vector
$m \ll n$	the magnitude of m is much, much less than the magnitude of n

All of these initiatives will require a refinement of the *Rules of the Air* by which aircraft manage proximity and by which collision risk is minimised. Such refinement needs to be based on a much improved understanding of the required optimal behaviour when aircraft are manoeuvring in close proximity. Rigorous models are therefore required to consolidate and extend the physical and mathematical foundation on which the present *Rules of the Air* are based and on which new rules may be constructed. Further background is provided in Ref. [21] as to the international development in ATM systems, and as to the engineering processes required based on both rigorous mathematics and formal methods. The most significant observation is that in the design of real, complex engineering systems it is not always possible to achieve the goal of finding closed-form solutions to a problem but the engineering imperative is always to seek to do so. We pursue this imperative in this paper.

A more precise specification of the location of the point of closest approach (PCA) between the trajectories of two vehicles, together with its associated evaluation of a miss distance, D, are important and necessary requirements for managing close proximity and for resolving conflict situations for both aircraft and ships (Refs. [22–32]). A common implementation has been to use D_{SS} the estimated minimum 3D miss-distance between the linear extrapolations of the instantaneous velocity vectors of two vehicles. This specific measure has had long standing within the aerospace industry (Refs. [3,4,23,27,33–37]). It is also identified within the international literature as the basis of various guidance laws (Refs. [23,27,34,35]).

However, in some situations D_{SS} may not be the appropriate measure to use. Such a case is during the simultaneous arrival of two aircraft to parallel runways. One aircraft can be on a left base for the left runway, the other on a right base for the right runway each flying a mirror-symmetric flight-path to the other. In this situation D_{SS} is zero for some duration in the turns yet operations are normal. Similar situations arise within the circuit area where aircraft turn circles are established in close proximity by well defined and prescribed spatial procedures. In these circumstances pilots need to be able to dynamically predict the miss distance, D_{TT}, between both aircraft while turning, and where this will occur in the turn, PCA_{TT} (not PCA_{SS}). This measure has been the subject of studies on the optimal resolution of both ship and of aircraft conflicts (Refs. [22–32]). While considerable advances based on numerical optimization techniques have been made making it possible to study complex scenarios involving many participants, the fact remains that the underlying mathematical essence has not be fully recovered. It is therefore important that such results be validated against analytic or semi-analytic syntheses of optimal solutions for realistic scenarios. To this end the research of this paper refines and elaborates the specification of the termination conditions found in Refs. [3,4,36], by removing the constraints of both a unity speed ratio and a unity turn-rate ratio thus augmenting the rigorous specification of termination conditions as found in Ref. [21]. Four fundamental cases for the termination criteria are identified and then examined with the intent of contributing to a suite of mathematical models that will more adequately support the assessment of performance and the validation of both the present *Rules of the Air* and of future proximity management procedures and systems.

1.1. Domain of the problem

The situation where two aircraft are both turning in close proximity is considered. The specific problem is to specify the nature of the spatial termination criteria under which PCA_{TT} occurs for the planar, non-intersecting turn-centric mode.