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Effect of rotation on the onset of double diffusive convection in a Darcy porous medium saturated with a couple stress fluid

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ABSTRACT

The effect of rotation on the onset of double diffusive convection in a horizontal couple stress fluid-saturated porous layer, which is heated and salted from below, is studied analytically using both linear and weak nonlinear stability analyses. The extended Darcy model, which includes the time derivative and Coriolis terms, has been employed in the momentum equation. The onset criterion for stationary, oscillatory and finite amplitude convection is derived analytically. The effect of Taylor number, couple stress parameter, solute Rayleigh number, Lewis number, Darcy–Prandtl number, and normalized porosity on the stationary, oscillatory, and finite amplitude convection is shown graphically. It is found that the rotation, couple stress parameter and solute Rayleigh number have stabilizing effect on the stationary, oscillatory, and finite amplitude convection. The Lewis number has a stabilizing effect in the case of stationary and finite amplitude modes, with a destabilizing effect in the case of oscillatory convection. The Darcy–Prandtl number and normalized porosity advances the onset of oscillatory convection. A weak nonlinear theory based on the truncated representation of Fourier series method is used to find the finite amplitude Rayleigh number and heat and mass transfer. The transient behavior of the Nusselt number and Sherwood number is investigated by solving the finite amplitude equations using Runge–Kutta method.

1. Introduction

The problem of double diffusive convection in porous media has attracted considerable interest during the last few decades because of its wide range of applications, from the solidification of binary mixtures to the migration of solutes in water-saturated soils. Other examples include geophysical systems, electro-chemistry, and the migration of moisture through air contained in fibrous insulation. The problem of double diffusive convection in a porous medium has been extensively investigated and the growing volume of work devoted to this area is well documented by Ingham and Pop [1,2], Nield and Bejan [3], Vafai [4,5] and Vadasz [6].

Although the problem of double diffusive convection has been extensively investigated for Newtonian fluids, relatively little attention has been devoted to this problem with non-Newtonian fluids. The corresponding problem in the case of a porous medium has also not received much attention until recently. With growing importance of non-Newtonian fluids with suspended particles in modern technology and industries, the investigations of such fluids are desirable. The study of such fluids have applications in a number of processes that occur in industry, such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubrication and colloidal and suspension solutions. These fluids

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deform and produce a spin field due to the microrotation of suspended particles forming micropolar fluid developed by Eringen [7]. The micropolar fluids take care of local effects arising from microstructure and as well as the intrinsic motions of microfluidics. The spin field due to microrotation of freely suspended particles set up an anti-symmetric stress, known as couple stress, and thus forming couple stress fluid. Thus, couple stress fluid, according to Eringen [7], is a particular case of micropolar fluid when microrotation balances with the natural vorticity of the fluid. In the category of non-Newtonian fluids couple stress fluids have distinct features, such as polar effects. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant. The constitutive equations for couple stress fluids were given by Stokes [8]. The theory proposed by Stokes is the simplest one for micro-fluids, which allows polar effects such as the presence of couple stress, body couple and non-symmetric tensors. There are few studies available on the Rayleigh–Benard problem for couple stress fluids, and extensions including the issue of stability/onset [9–19].

The study of the effect of external rotation on thermal convection has attracted significant experimental and theoretical interest. Because of its general occurrence in geophysical and oceanic flows, it is important to understand how the Coriolis force influences the structure and transport properties of thermal convection. Rotating thermal convection also provides a system to study hydrodynamic instabilities, pattern formation and spatiotemporal chaos in nonlinear dynamical systems. The study of thermal convection in rotating porous media is motivated both theoretically and by its practical applications in engineering some of the important areas of applications in engineering include the food processing, chemical process, solidification and centrifugal casting of metals and rotating machinery. During the last two decades there has been a great deal of effort lead by many researchers on the study of effect of external rotation on the Rayleigh–Benard convection. In the literature there are plenty of works available on the problem of understanding how the Coriolis force influences the onset of thermal convection. The linear dynamics of rotating Rayleigh–Benard convection with rigid stress-free boundaries has been thoroughly investigated by Chandrasekhar [20] who determined the marginal stability boundary and critical horizontal wave numbers for the onset of convection and over stability as a function of the Taylor number. Vadasz [21] has used linear and weak nonlinear stability theories to study the effect of Coriolis force on gravity-driven convection in a rotating porous layer heated from below by employing the modified Darcy model. The differences as well as similarities between the porous medium and pure fluids convection results are highlighted in this study. An excellent review of research on thermal convection in a rotating porous medium is given by Vadasz [22]. A nonlinear stability analysis for thermal convection in a rotating porous layer has been performed by Straughan [23]. Chakrabarti and Gupta [24] have analyzed the nonlinear thermohaline convection in a rotating porous medium. Govender [25] has analyzed the effect of Coriolis force on centrifugally driven convection in a rotating layer of porous medium. Malashetty et al. [26] have studied linear and nonlinear thermal convection in a rotating porous layer using a thermal nonequillibrium model. Shivakumara et al. [27] have investigated the effect of Coriolis force on thermal convection in a layer of Newtonian fluid-saturated porous medium using the Brinkman–Lapwood–Darcy model with fluid viscosity different from Brinkman viscosity. The effect of rotation on the onset of double diffusive convection in a horizontal anisotropic porous layer was studied by Malashetty and Heera [28]. Convective instability in either a couple stress fluid layer or couple stress fluid-saturated porous layer heated from below has been investigated in the recent past including the effects of an additional diffusing component (i.e., solute concentration) and external constraints such as magnetic field and /or rotation.

Goel et al. [29] have studied the hydromagnetic stability of an unbounded couple stress binary fluid mixture under rotation with vertical temperature and solute concentration gradients. A layer of couple stress fluid saturating a porous medium heated from below in the presence of rotation has been studied by Sharma et al. [30], and condition for the onset of convection is obtained. Sunil et al. [31] have investigated the effect of magnetic field and rotation on a layer of couple stress fluid heated from below in a porous medium. Sharma and Sharma [32] have investigated the effect of suspended particles on electrically conducting couple stress fluid heated uniformly from below under the influence of uniform rotation and magnetic field. Recently, Shivakumara et al. [33] has discussed the Coriolis effect on thermal convection in a couple stress fluid-saturated rotating rigid porous layer. Further, the effect of rotation on the onset of double diffusive convection in a couple stress fluid-saturated porous medium is not available. The intent of the present paper is therefore to study the onset of double diffusive convection in a couple stress fluid-saturated rotating porous layer heated and salted from below using linear and weak nonlinear analyses with emphasis on how the condition for the onset of convection is modified in the presence of rotation and couple stresses.

2. Mathematical formulation

We consider an infinite horizontal fluid-saturated porous layer confined between the planes $z = 0$ and $z = d$, with the vertically downward gravity force $\mathbf{g}$ acting on it. The temperatures T_l and T_u with $T_l > T_u$ and solute concentrations S_l and S_u with $S_l > S_u$ are imposed at the bottom and top boundaries, respectively. The boundaries are impermeable, and we assume that the fluid and solid phases are in local thermal equilibrium. A Cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis vertically upwards. The porous layer rotates uniformly about the z-axis with a constant angular velocity $\mathbf{\Omega} = (0, 0, \Omega)$. The interaction between heat and mass transfer, known as Soret and Dufour effects, is supposed to have no influence on the convective flow, so they are ignored. The velocities are assumed to be small so that the advective and Forchheimer inertia effects are ignored. The flow in the porous medium is governed by the modified Darcy's law, which includes the time derivative and the Coriolis terms is employed as a momentum equation.