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A circular global profit Malmquist productivity index in data envelopment analysis

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ABSTRACT

To remove the difficulty caused by different profit frontiers in different periods of time for calculating profit efficiency changes and its components, this paper proposes a circular global profit Malmquist productivity index. This index is applicable when the input costs and output prices are known and when producers seek to maximize the total profit of their decision making units (DMUs). To this end, first, two methods are introduced to obtain the common costs and prices with or without the decision maker's preferences, and then, a common profit efficient frontier is obtained. The proposed index can be decomposed into several circular components, viz., profit efficiency change, profit technical change, technical efficiency change, allocative efficiency change, technical change, and cost/price change. The proposed index is then generalised to compare the productivity of two different units at two different points in time. The global profit Malmquist productivity index developed here is unique and is computed using nonparametric linear programming model known as data envelopment analysis (DEA), and there is no need to resort to the geometric mean in the calculation. To illustrate the proposed index and its components, numerical examples at three successive periods of time are given.

1. Introduction

Data envelopment analysis (DEA) is a nonparametric method that can be used to evaluate the productivity changes of a decision making unit (DMU) with multiple inputs and outputs. To calculate the productivity changes of a DMU at different periods of time, Caves et al. [1] proposed a Malmquist productivity Index (MI). MI has many applications [2–6]. In particular, it can be decomposed into technical efficiency change and technical change components. In this framework, Maniadakis and Thanassoulis [7] modified the Malmquist index to measure the productivity change when the input prices are known and when the producer wants to minimize the costs.

Pastor and Lovell [8] proposed a circular global Malmquist productivity index, which gives a single measure of productivity change. This index is based on technology that takes into account the data of all producers for all periods. Also, Portela and Thanassoulis [9] proposed a meta-Malmquist index under the Constant Returns to Scale (CRS) and Variable Returns to Scale (VRS) technologies. This index can be decomposed into the circular components of efficiency and technical change.

The cost Malmquist index proposed by Maniadakis and Thanassoulis [7] was extended by Tohidi et al. [10] into the Profit Malmquist index (PM), which can be used when input and output prices are available and when the producers want to maximize the total profit of DMUs. To compare the cost efficiency of DMUs for the different periods of time, Tohidi et al. [11] used the convex combination (weighted average) of the inputs' costs for different periods of time and obtained a common cost for

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the inputs. They used the common cost and obtained a global cost efficient frontier as the single base and proposed a global cost Malmquist index. This index is circular and gives a single measure of productivity change, and its models are always feasible.

Common (share) costs, prices, and weights have been used in the economy and in DEA [12,13]. For instance, Werner and Thaler [14] used the weighted average (convex combination) as a cost in stock marketing. In regression analysis and in economic surveys of time series analysis, the share costs and prices have been used [15,16].

Kao [12] used common weights (instead of time periods weights) and compared the technical efficiency of DMUs for different time periods and introduced a global frontier as the single base by using the common weights of all of the time periods.

There are several profit-efficient frontiers due to the different costs and prices for different periods of time. To remove the difficulty caused by different profit frontiers in calculating profit efficiency changes and profit efficiency components changes, under the CRS and VRS assumptions, the present paper suggests a new PM index referred to as the global Profit Malmquist productivity index (PM^G). It can be used when the input costs and output prices are available. Toward this end, this paper uses Multi-Objective Programming (MOP) and weighted average methods [17,18] and proposes two methods with or without decision maker preferences to obtain the common costs and prices as the coefficients of the base profit efficient frontier. It can be considered as a generalised form of Tohidi et al.'s method of dealing with common costs as inputs [11].

The proposed PM^G index and its components (i.e., 1. profit efficiency change, 2. profit technical change, 3. technical efficiency change, 4. allocative efficiency change, 5. technical change, and 6. cost and price change) satisfy the circularity property and generate a single measure of profit change without needing to resort to the geometric mean in the calculation. Then, the index is generalised to compare the productivity of the two different units at two different points in time.

The paper has the following structure. Section 2 presents a background of the PM and meta-Malmquist indices. Section 3 proposes two methods to obtain the common costs and prices as coefficients of the profit efficient frontier. A global profit Malmquist productivity index is introduced in section 4, and then, the proposed index is decomposed, and its components are presented under the CRS and VRS assumptions. Section 5 compares the productivity of two different units at two different points of time. In section 6, we calculate the index and its components. Section 7 illustrates the proposed global profit Malmquist index and its components using a numerical example at three different periods of time, and then verifies their properties. Finally, section 8 concludes the paper.

2. Background

For a time period t ($t = 1,\ldots,T$), let DMU_j ($j = 1,\ldots,n$) use inputs $x_j^t = \left(x_{1j}^t,\ldots,x_{mj}^t\right) \in R_+^m$ to produce outputs $y_j^t = \left(y_{1j}^t,\ldots,y_{sj}^t\right) \in R_+^s$. Also, the output price vector $p^t \in R_+^s$ and the input cost vector $c^t \in R_+^m$ are available. The profit Malmquist productivity index (PM) [10] of periods $t\left(\mathrm{PM}_j^t\right)$ and $t+1\left(\mathrm{PM}_j^{t+1}\right)$ and their geometric mean (PM_j) for DMU_j with the coordinate $\left(x_j^t, y_j^t\right)$ are as follows, respectively.

$$\mathrm{PM}_j^t = \left[\frac{\dfrac{PR^t\left(x_j^{t+1},y_j^{t+1},c^t,p^t\right)}{\frac{p^t y_j^{t+1}}{c^t x_j^{t+1}}}}{\dfrac{PR^t\left(x_j^t,y_j^t,c^t,p^t\right)}{\frac{p^t y_j^t}{c^t x_j^t}}}\right], \quad \mathrm{PM}_j^{t+1} = \left[\frac{\dfrac{PR^{t+1}\left(x_j^{t+1},y_j^{t+1},c^{t+1},p^{t+1}\right)}{\frac{p^{t+1} y_j^{t+1}}{c^{t+1} x_j^{t+1}}}}{\dfrac{PR^{t+1}\left(x_j^t,y_j^t,c^{t+1},p^{t+1}\right)}{\frac{p^{t+1} y_j^t}{c^{t+1} x_j^t}}}\right] \quad \text{and} \quad \mathrm{PM}_j = \left[\frac{\dfrac{PR^t\left(x_j^{t+1},y_j^{t+1},c^t,p^t\right)}{\frac{p^t y_j^{t+1}}{c^t x_j^{t+1}}}}{\dfrac{PR^t\left(x_j^t,y_j^t,c^t,p^t\right)}{\frac{p^t y_j^t}{c^t x_j^t}}} \times \frac{\dfrac{PR^{t+1}\left(x_j^{t+1},y_j^{t+1},c^{t+1},p^{t+1}\right)}{\frac{p^{t+1} y_j^{t+1}}{c^{t+1} x_j^{t+1}}}}{\dfrac{PR^{t+1}\left(x_j^t,y_j^t,c^{t+1},p^{t+1}\right)}{\frac{p^{t+1} y_j^t}{c^{t+1} x_j^t}}}\right]^{1/2},$$

where $PR^t\left(x_j^t, y_j^t, c^t, p^t\right) = \max\left\{p^t y^t / c^t x^t : (x^t, y^t) \in T_C^t, c^t > 0, p^t > 0\right\}$ is the technology in terms of the profit function [19], and T_C^t is the production technology of the period t ($t = 1,\ldots,T$), and the subscript "C" indicates that the production technology satisfies the CRS property. Using $PR^t\left(x_j^t, y_j^t, c^t, p^t\right)$, the set $\overline{PR}^t = \left\{(x^t, y^t) : p^t y^t / c^t x^t = PR^t\left(x_j^t, y_j^t, c^t, p^t\right)\right\}$ is defined as the profit boundary of period t ($t = 1,\ldots,T$) [8].

Portela and Thanassoulis [9] proposed a circular meta-Malmquist index to measure productivity. They obtained the productivity change of unit j between periods t and $t+1$ in the following manner:

$$\mathrm{MI}_{t,t+1}^j = \frac{\theta_{jt+1}^m}{\theta_{jt}^m}.$$

θ_{jt}^m is the meta-efficiency of unit j as observed in period t and is obtained using the following model [9]:

$$\begin{aligned}
\theta_{j_o\tau}^m = \min\ & k_{j_o} \\
\text{s.t.}\ & \sum_{t=1}^{T}\sum_{j=1}^{n} \lambda_{jt} x_{ij}^t \leqslant k_{j_o} x_{ik}^\tau, \quad i = 1,\ldots,m, \\
& \sum_{t=1}^{T}\sum_{j=1}^{n} \lambda_{jt} y_{rj}^t \geqslant y_{rk}^\tau, \quad r = 1,\ldots,s, \\
& \lambda_{jt} \geqslant 0, j = 1,\ldots,n, \quad t = 1,\ldots,T, k_{j_o}\ \mathit{free}
\end{aligned}$$