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Asymmetric free vibration of circular plate in contact with incompressible fluid

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ABSTRACT

Vibration of circular plates in contact with fluid has extensive applications in the industry. This paper derives added mass and frequencies for asymmetric free vibration of coupled system including clamped circular plate in contact with incompressible bounded fluid. Considering small oscillations induced by the plate vibration in the incompressible and inviscid fluid, velocity potential function is used to describe the fluid motion. Derivation uses Kirchoff's thin plate theory. Two approaches are used to derive the free vibration frequency of the system. The solutions include an analytical solution employing Fourier–Bessel series and a variational formulation applied simultaneously on the plate and fluid. Strong correlation is found between free vibration frequencies of the two solutions. Finally the effect of fluid depth on the added mass and free vibration frequencies of the coupled system is investigated.

1. Introduction

Circular plates in contact with fluid have extensive application in engineering as micro pumps, coolant in integral-type nuclear reactor, circular disk of butterfly valves, etc. The dynamic characteristic of the plate in contact with fluid (wet plate) is different from that of plate alone (dry plate). A portion of the fluid that oscillates with the plate by increasing the mass of the coupled system decreases the natural free vibration frequency of the wet plate. This will affect the performance of the coupled system under dynamic loading.

Theoretical and experimental studies on the natural frequencies of free-edge annular plates resting on free surface or completely submerged were carried out by Kwak and Amabili [1]. Considering an unbounded fluid domain, they introduced nondimensionalized added virtual mass incremental (NAVMI) factors in order to estimate the fluid effect on the individual natural frequencies of the wet plate. Accounting for surface waves Amabili and Kwak [2] derived the response of circular plate on free fluid surface. They used perturbation method to decompose the mixed boundary condition on the free surface to two separate problems involving very high and very low frequency problems. Robinson and Palmer [3] carried out a modal analysis of thin plate floating on the surface of liquid and oscillating at low frequencies and derived governing equation for the coupled system. Ginsberg and Chu [4] developed a variational platform to derive the mode shapes of plate in contact with heavy fluid. Esmailzadeh et al. [5] derived the free vibration frequencies of structural elements containing and or submerged in fluid. They used potential function to calculate hydrodynamic fluid pressure on the structure. The fluid depth has important effect on the interaction between fluid and structure. Kwak and Han [6] investigated the effect of the fluid depth on the vibration of free edge circular plate in contact with fluid. Considering the response of the system in high frequency

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range, they ignored the effect of the surface waves. Using Hankel transformation, they derived a dual integral equation. Jeong and Kim [7] considered the hydroelastic vibration of a circular plate submerged in bounded compressible fluid. Accounting for compatibility of deflection between plate and fluid, they used Fourier–Bessel series to solve the dynamic equilibrium equation. Jeong [8] derived the added mass and free vibration frequencies for two identical circular plates coupled by bounded fluid. Espinosa and Gallego-Juarez [9] studied the vibration of plates submerged in water paying attention to the lower modes using analytical and experimental methods. Zhou and Cheung [10] analyzed the hydroelastic vibration of vertical rectangular plate in contact with water. Cho et al. [11] studied the modal characteristics of a liquid storage tank baffled with annular plate using coupled structural–acoustic finite element method. Ostasevicius et al. [12] accounting for viscous air damping studied the free and forced vibration response of a micro-beam. They evaluated the response considering linear and nonlinear forms of Reynolds equation. Harrison et al. [13] investigated the response of a plate oscillating near a rigid boundary accounting for squeeze film-damping effect. They showed experimentally that resonance frequency increases as the distance from the rigid boundary increases.

This paper investigates the free vibration of clamped circular plate in contact with incompressible bounded fluid. Due to incompressibility of the plate, there is no possibility for axisymmetric vibration of the plate. Considering the compatibility of the deflections and using the Fourier–Bessel series, this paper derives added mass and frequencies for asymmetric free vibration of the coupled system. The free vibration frequencies of the system are compared with those derived employing variational formulation. Finally the effect of the fluid depth on the system's free vibration response is evaluated.

2. Derivation using Fourier–Bessel series

Fig. 1 depicts a circular plate placed over a bounded incompressible and inviscid fluid. Dynamic equilibrium equation of the plate in contact with fluid is

$$D\nabla^4 w + \rho \ddot{w} = P, \tag{1}$$

where w is the plate deflection, D is bending stiffness of the plate, ρ the plate density and P is the fluid pressure. In the case of incompressible fluid contained in the rigid vessel, there will not be any contribution from axisymmetric modes in the overall response. Considering the asymmetric modes, the free vibration mode shapes of clamped circular plate in the air (dry plate) can be written as [14]

$$w_{ij} = \left[I_i(\lambda_{ij}) J_i\left(\frac{\lambda_{ij} r}{a}\right) - J_i(\lambda_{ij}) I_i\left(\frac{\lambda_{ij} r}{a}\right)\right] \cos(i\theta), \quad i,j = 1,2,\ldots, \tag{2}$$

where j is the number of diametrical nodes, i is the number of circular nodes, w_{ij} is mode shape associated with ith circular node and jth diametrical nodes, I_i and J_i are Bessel functions of first and second kind of order one, and λ_{ij} is the frequency parameter. Imposing boundary conditions of the clamped plate, the frequency parameter could be obtained by solving the characteristic equation

$$J_{i-1}(\lambda_{ij}) I_i(\lambda_{ij}) - J_i(\lambda_{ij}) I_{i-1}(\lambda_{ij}) = 0, \quad i,j = 1,2,\ldots \tag{3}$$

Therefore the plate displacement could be rewritten as

$$w(r,\theta,t) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\left[I_i(\lambda_{ij}) J_i\left(\frac{\lambda_{ij} r}{a}\right) - J_i(\lambda_{ij}) I_i\left(\frac{\lambda_{ij} r}{a}\right)\right] \cos(i\theta) A_{ij}(t) = \sum_{k=1}^{\infty} w_k(r)\cos(i\theta) A_k(t) = \sum_{k=1}^{\infty} \bar{w}_k(r,\theta) A_k(t), \tag{4}$$

where A_k and w_k are modal amplitude and mode shape corresponding to a specific pair of i and j. In other word, a one to one correspondence between k and pair of (i,j) is assumed.

Considering the free vibration of the plate, Eq. (1) takes the following form

$$\nabla^4 \bar{w}_k - \lambda_k^4 \bar{w}_k = 0, \quad \lambda_k^4 = \frac{\omega_k^2 \rho}{D}. \tag{5}$$

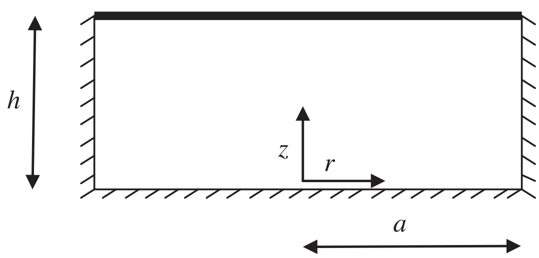

Fig. 1. Clamped circular plate in contact with fluid.