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Optimal replacement model with age-dependent failure type based on a cumulative repair-cost limit policy

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ABSTRACT

This paper presents a replacement model with age-dependent failure type based on a cumulative repair-cost limit policy, whose concept uses the information of all repair costs to decide whether the system is repaired or replaced. As failures occur, the system experiences one of the two types of failures: a type-I failure (minor), rectified by a minimal repair; or a type-II failure (catastrophic) that calls for a replacement. A critical type-I failure means a minor failure at which the accumulated repair cost exceeds the pre-determined limit for the first time. The system is replaced at the nth type-I failure, or at a critical type-I failure, or at first type-II failure, whichever occurs first. The optimal number of minimal repairs before replacement which minimizes the mean cost rate is derived and studied in terms of its existence and uniqueness. Several classical models in maintenance literature are special cases of our model.

1. Introduction

Almost all systems deteriorate with age and usage and are subject to stochastic failures during operation. Furthermore, consecutive failures or catastrophic breakdowns are dangerous and costly to a system, it becomes great importance to reduce operating costs and avoid the risk of a catastrophic breakdown. Therefore, determining an optimal replacement policy for a system is a major research in reliability literatures.

Barlow and Proschan [1] presented the traditional age-replacement maintenance policy which a system is replaced at a failure or at age T, whichever occurs first. Several extensions of this policy have been investigated, such as [2–8]. Furthermore, Boland and Proschan [9] considered the case of periodic replacement at times kT $(k = 1, 2, \ldots)$ and minimal repair if the system fails otherwise. This model has been extended by [10–18], among others. Besides, Makabe and Morimura [19–21] proposed a replacement model where a system is replaced at the nth failure, and they also discussed the determination of the optimum policy. This model has been evolved by [3,7,10,17,22–25].

The system restores its functioning condition just prior to failure via a minimal repair. A repair-cost limit policy with minimal repair, which prescribe the repair or replace decision depending on one single repair cost, has been first discussed by Drinkwater and Hastings [26]. In such a repair-cost limit policy, if the repair cost exceeds a certain threshold, the system is replaced rather than repaired. Several extensions of this policy have been investigated in [2,4,5,27–35]. The primary deficiency of the previous repair-cost limit policy is that the repair/replace decision is based simply on the cost of one single repair, even a system with frequent but not-very-costly failures and consequently high accumulated repair costs will

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continue to be repaired rather than replaced. Obviously, making the repair/replace decision based on entire repair-cost history seems more reasonable. Beichelt [31] presented an improved replacement policy based on the repair-cost rate limit: the system is replaced as soon as the repair-cost rate exceeds a threshold level. Beichelt [36] further proposed a cumulative repair-cost limit replacement policy which requires the system be replaced as soon as the accumulated maintenance cost $C(t)$ reaches or exceeds a given limit, but $C(t)$ was given exogenously and was not based on the repair history.

This paper presents a generalized model for determining the optimal replacement policy based on multiple factors such as the number of minimal repairs before replacement, and the cumulative repair-cost limit. The decision to repair or replace a system at minor failures depends on the probability of failure types and the accumulated repair costs. The main characteristic of our model is that the self-cumulative repair-cost viewpoint (entire repair-cost history) is considered. In fact, if repairable failures occur at random point, then the random repair costs should be accumulated additively to a system. The accumulated repair costs need not to be given beforehand, but are endogenous through applying information from the system's entire repair-cost history. As such the expected value of the total repair cost could be evaluated from the failure process and the self-cumulative repair-cost. Such a stochastic model generates a cumulative process. The similar aspect about cumulative damage process was discussed by [37,38].

The rest of the paper is organized as follows: Section 2 presents the model formulation and optimization. Section 3 shows that several classical maintenance models are special cases of our proposed model. Section 4 develops an algorithm for determining the optimal policy parameter, and a computational example is provided to demonstrate the use of the algorithm. Section 5 concludes.

2. General model

In the replacement policy, the planned (scheduled) replacement occurs whenever the number of repairable (minor) failures reaches a threshold value n, and the unplanned replacement occurs at the kth minor failure at which the accumulated repair cost exceeds the pre-determined limit L or at the first catastrophic failure. A replacement cycle of the system is defined as the time interval between installation and first replacement or between two consecutive replacements. In this framework, replacement cycles constitute a regenerative process. Below is a list of notations used in this paper.

Notations

X	time to failure of a new system
$f(\cdot), F(\cdot)$	probability density function (PDF), cumulative distribution function (CDF) of X
$\bar{F}(\cdot)$	survival function (SF) of X; $\bar{F}(\cdot) = 1 - F(\cdot)$
$r(t)$	failure (hazard) rate function of X; $r(t) = f(t)/\bar{F}(t)$
$\Lambda(t)$	cumulative hazard function of X; $\Lambda(t) = \int_0^t r(x)\mathrm{d}x = -\ln \bar{F}(t)$
$p(t)$	Pr{a type-II failure when the system fails at age t}
$q(t)$	Pr{a type-I failure when the system fails at age t}; $q(t) = 1 - p(t)$
Y	waiting time until first type-II failure
$F_p(\cdot), \bar{F}_p(\cdot)$	CDF, SF of Y; $\bar{F}_P(\cdot) = 1 - F_p(\cdot)$
$\{N_1(t) : t \geqslant 0\}$	non-homogeneous Poisson process (NHPP) with intensity function $q(t)r(t)$
S_j	waiting time until the jth type-I failure for $j = 1, 2, 3, \ldots$; $S_0 = 0$
$f_{S_j\mid Y}(\cdot)$, $F_{S_j\mid Y}(\cdot)$	conditional PDF and CDF of S_j
W_i	minimal repair cost due to the ith type-I failure for $i = 1, 2, 3, \ldots$
$g(\cdot)$, $G(\cdot)$	PDF, CDF of W_i
c_w	mean cost of W_i; $c_w = E[W_i]$
Z_j	accumulated repair cost until the jth type-I failure for $j = 1, 2, 3, \ldots$; $Z_j = \sum_{i=1}^{j} W_i$
$G^{(j)}(z)$	CDF of Z_j; the j-fold Stieltjes convolution of the distribution G with itself
n	number of minimal repairs before replacement; $n > k$
L	total repair-cost limit
c_0	cost of a planned replacement
c_1	cost of a critical type-I failure replacement
c_2	cost of a type-II failure replacement
U_i	length of the ith replacement cycle for $i = 1, 2, \ldots$
V_i	operational cost over U_i
$D(t)$	s-expected cost of the operating system over $[0, t]$
$C(n)$	s-expected cost-rate for an infinite time span; $C(n) = E[V_1]/E[U_1]$
n^*	n which minimizes $C(n)$

Additional symbols are defined as needed.