Applied Mathematical Modelling 37 (2013) 318–327

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Same families of geometric aggregation operators with intuitionistic trapezoidal fuzzy numbers

Jian Wu *, Qing-wei Cao

School of Economics and Management, Zhejiang Normal University, Box 62 at YingBinDaDao 688, Jinhua 321004, Zhejiang Province, PR China

A R T I C L E   I N F O

Article history:
Received 27 August 2011
Received in revised form 15 February 2012
Accepted 1 March 2012
Available online 7 March 2012

Keywords:
Group decision making
Multiple attribute decision making
Intuitionistic trapezoidal fuzzy numbers
Aggregation operation

A B S T R A C T

The aim of this work is to present some cases of aggregation operators with intuitionistic trapezoidal fuzzy numbers and study their desirable properties. First, some operational laws of intuitionistic trapezoidal fuzzy numbers are introduced. Next, based on these operational laws, we develop some geometric aggregation operators for aggregating intuitionistic trapezoidal fuzzy numbers. In particular, we present the intuitionistic trapezoidal fuzzy weighted geometric (ITFWG) operator, the intuitionistic trapezoidal fuzzy ordered weighted geometric (ITFOWG) operator, the induced intuitionistic trapezoidal fuzzy ordered weighted geometric (I-ITFOWG) operator and the intuitionistic trapezoidal fuzzy hybrid geometric (ITFHG) operator. It is worth noting that the aggregated value by using these operators is also an intuitionistic trapezoidal fuzzy value. Then, an approach to multiple attribute group decision making (MAGDM) problems with intuitionistic trapezoidal fuzzy information is developed based on the ITFWG and the ITFHG operators. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

1. Introduction

Atanassov [1] introduced the concept of intuitionistic fuzzy set (IFS) characterized by a membership function and a non-membership function, which is a generalization of the concept of fuzzy set [2]. The intuitionistic fuzzy set theory has been applied to many different fields, such as decision making [3–11], supplier selection [12,13], investment option [14], and logic programming [15,16]. But, most of these techniques use the minimum and maximum operations to carry the combination process, which usually produces the consequent loss of information and hence a lack of precision in the final results. To overcome this limitation, Xu [17] developed the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator, and the intuitionistic fuzzy hybrid aggregation (IFHA) operator.

Recently, Shu et al. [18] gave the definition and operational laws of intuitionistic triangular fuzzy number. A prominent characteristic of the intuitionistic triangular fuzzy set is that its domain is a consecutive set. Then the intuitionistic triangular fuzzy set constitutes an extension of Atanassov's fuzzy set intuitionistic fuzzy set (IFS) in which the domain is a discrete set. Same authors had paid attention to the intuitionistic triangular fuzzy set. Wang [19] gave the definition of intuitionistic trapezoidal fuzzy number and interval intuitionistic trapezoidal fuzzy number. Further, Wang and Zhang [20] developed the Hamming distance of intuitionistic trapezoidal fuzzy numbers and the intuitionistic trapezoidal fuzzy weighted arithmetic averaging (ITFWAA) operator. Wei [21] proposed some arithmetic aggregation operators including the intuitionistic trapezoidal fuzzy ordered weighted averaging (ITFOWA) operator and the intuitionistic trapezoidal fuzzy hybrid aggregation

* Corresponding author. Tel.: +86 579 82298567.
E-mail address: jyajian@163.com (J. Wu).

0307-904X/$ - see front matter © 2012 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.apm.2012.03.001

(ITFHA) operator. But, the knowledge on the use of these aggregation operators for intuitionistic trapezoidal fuzzy numbers is still quite limited. For example, we still will ask the following questions:

(1) If $\tilde{\alpha}_i = ([a_i, b_i, c_i, d_i]; \mu_{\tilde{\alpha}_i}, v_{\tilde{\alpha}_i})$ ($i = 1, 2, \ldots, n$) is a collection of intuitionistic trapezoidal fuzzy numbers, is the aggregated value by using these operators also an intuitionistic trapezoidal fuzzy number?
(2) Are there some new aggregation operators for aggregating intuitionistic trapezoidal fuzzy numbers?

The aim of this article is to develop some familiars of geometric aggregation operators with intuitionistic trapezoidal fuzzy numbers including the intuitionistic trapezoidal fuzzy weighted geometric (ITFWG) operator, the intuitionistic trapezoidal fuzzy ordered weighted geometric (ITFOWG) operator, the induced intuitionistic trapezoidal fuzzy ordered weighted geometric (I-ITFOWG) operator and the intuitionistic trapezoidal fuzzy hybrid geometric (ITFHG) operator. The novelty of these aggregation operators is that the aggregated value by using these operators is also an intuitionistic trapezoidal fuzzy value.

In order to do that, this work is set out as follows. Section 2 briefly reviews some basic concepts and operation laws related to intuitionistic trapezoidal fuzzy numbers. Section 3 defines the concept of the ITFWG, the ITFOWG, the I-ITFOWG and the ITFHG operators and studies their desirable properties. In Section 4, an ITFWG and ITFHG operators based approach to multiple attribute group decision making (MAGDM) problems with intuitionistic trapezoidal fuzzy information is presented. Section 5 provides an illustrative example. Finally, in Section 6 we draw our conclusions.

2. Preliminaries

We start this section by introducing some basic concepts related to intuitionistic trapezoidal fuzzy numbers, which will be used throughout this paper.

Wang and Zhang [20] gave the definition of intuitionistic trapezoidal fuzzy number and some operational laws of intuitionistic trapezoidal fuzzy numbers as follows:

Definition 1 [20]. Let $\tilde{\alpha}$ is an intuitionistic trapezoidal fuzzy number, its membership function is:

$$\mu_{\tilde{\alpha}}(x) = \begin{cases} \frac{x-a}{b-a}\mu_{\tilde{\alpha}}, & a \leqslant x < b; \\ \mu_{\tilde{\alpha}}, & b \leqslant x \leqslant c; \\ \frac{d-x}{d-c}\mu_{\tilde{\alpha}}, & c < x \leqslant d; \\ 0, & \text{others} \end{cases} \tag{1}$$

its non-membership function is:

$$v_{\tilde{\alpha}}(x) = \begin{cases} \frac{b-x+v_{\tilde{\alpha}}(x-a_1)}{b-a}, & a_1 \leqslant x < b; \\ v_{\tilde{\alpha}}, & b \leqslant x \leqslant c; \\ \frac{x-c+v_{\tilde{\alpha}}(d_1-x)}{d_1-c}, & c < x \leqslant d_1; \\ 0, & \text{others}, \end{cases} \tag{2}$$

where $0 \leqslant \mu_{\tilde{\alpha}} \leqslant 1;\ 0 \leqslant v_{\tilde{\alpha}} \leqslant 1;\ \mu_{\tilde{\alpha}} + v_{\tilde{\alpha}} \leqslant 1;\ a, b, c, d \in R$. Then $\tilde{\alpha} = \langle([a, b, c, d]; \mu_{\tilde{\alpha}}), ([a_1, b, c, d_1]; v_{\tilde{\alpha}})\rangle$ is called an intuitionistic trapezoidal fuzzy number. For convenience, let $\tilde{\alpha} = ([a, b, c, d]; \mu_{\tilde{\alpha}}, v_{\tilde{\alpha}})$.

Definition 2 [20]. Let $\tilde{\alpha}_1 = ([a_1, b_1, c_1, d_1]; \mu_{\tilde{\alpha}_1}, v_{\tilde{\alpha}_1})$ and $\tilde{\alpha}_2 = ([a_2, b_2, c_2, d_2]; \mu_{\tilde{\alpha}_2}, v_{\tilde{\alpha}_2})$ be two intuitionistic trapezoidal fuzzy numbers, and $\lambda \geqslant 0$, then

(1) $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = ([a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]; \mu_{\tilde{\alpha}_1} + \mu_{\tilde{\alpha}_2} - \mu_{\tilde{\alpha}_1}\mu_{\tilde{\alpha}_2}, v_{\tilde{\alpha}_1}v_{\tilde{\alpha}_2})$;
(2) $\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = ([a_1a_2, b_1b_2, c_1c_2, d_1d_2]; \mu_{\tilde{\alpha}_1}\mu_{\tilde{\alpha}_2}, v_{\tilde{\alpha}_1} + v_{\tilde{\alpha}_2} - v_{\tilde{\alpha}_1}v_{\tilde{\alpha}_2})$;
(3) $\lambda\tilde{\alpha} = ([\lambda a, \lambda b, \lambda c, \lambda d]; 1 - (1 - \mu_{\tilde{\alpha}})^{\lambda}, v_{\tilde{\alpha}}^{\lambda})$;
(4) $\tilde{\alpha}^{\lambda} = ([a^{\lambda}, b^{\lambda}, c^{\lambda}, d^{\lambda}]; \mu_{\tilde{\alpha}}^{\lambda}, 1 - (1 - v_{\tilde{\alpha}})^{\lambda})$.

Definition 3 [20]. Let $\tilde{\alpha}_1 = ([a_1, b_1, c_1, d_1]; \mu_{\tilde{\alpha}_1}, v_{\tilde{\alpha}_1})$ and $\tilde{\alpha}_2 = ([a_2, b_2, c_2, d_2]; \mu_{\tilde{\alpha}_2}, v_{\tilde{\alpha}_2})$ be two intuitionistic trapezoidal fuzzy numbers, and $\lambda \geqslant 0$, then the normalized Hamming distance between $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ is defined as follows:

$$\begin{aligned} d(\tilde{\alpha}_1, \tilde{\alpha}_2) = \frac{1}{8}(&|(1 + \mu_{\tilde{\alpha}_1} - v_{\tilde{\alpha}_1})a_1 - (1 + \mu_{\tilde{\alpha}_2} - v_{\tilde{\alpha}_2})a_2| + |(1 + \mu_{\tilde{\alpha}_1} - v_{\tilde{\alpha}_1})b_1 - (1 + \mu_{\tilde{\alpha}_2} - v_{\tilde{\alpha}_2})b_2| + |(1 + \mu_{\tilde{\alpha}_1} \\ &- v_{\tilde{\alpha}_1})c_1 - (1 + \mu_{\tilde{\alpha}_2} - v_{\tilde{\alpha}_2})c_2| + |(1 + \mu_{\tilde{\alpha}_1} - v_{\tilde{\alpha}_1})d_1 - (1 + \mu_{\tilde{\alpha}_2} - v_{\tilde{\alpha}_2})d_2|). \end{aligned} \tag{3}$$

Definition 4 [22]. For a normalized intuitionistic trapezoidal fuzzy decision making matrix $\widetilde{R} = (\tilde{r}_{ij})_{m\times n} = ([a_{ij}, b_{ij}, c_{ij}, d_{ij}]; \mu_{ij}, v_{ij})_{m\times n}$, where $0 \leqslant a_{ij} \leqslant b_{ij} \leqslant c_{ij} \leqslant d_{ij} \leqslant 1$, $0 \leqslant \mu_{ij}, v_{ij} \leqslant 1$, $0 \leqslant \mu_{ij} + v_{ij} \leqslant 1$, the intuitionistic trapezoidal fuzzy positive ideal solution and intuitionistic trapezoidal fuzzy negative ideal solution are defined as follows: