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Ground movement analysis in deep iron mine using fuzzy probability theory

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ABSTRACT

The analysis of the ground movements due to underground mining operation is one of the many important problems of rock mass mechanics. It is difficult to calculate the ground movement due to deep underground mining of iron ore accurately because of the complexity of the problems. In this paper, the application is described of the fuzzy probability measures to the analysis of ground movements. Based on the definition of the fuzzy probability measure, the theories for the two- and three-dimensional problems are developed and are applied to the analysis of ground movements duo to underground deep underground mining of iron ore.

1. Introduction

Any underground engineering excavation (such as underground mining, railway, underground storehouses, etc.) will certainly result in rock mass displacements and ground movements of varying degrees [1]. The ground movements due to excavation operations especially mining of ore have often resulted in major disasters occurring throughout the world, frequently with considerable loss of life and damage to property.

In underground and surface mining, if a void is excavated in a rock continuum, load formerly carried on the rock in the opening will be transferred either to the rock surrounding the opening or to supports (pillars) within the opening, or both, and finally to the ground surface. This results in a macroscopically nonuniform deformation of the surface in the horizontal or vertical direction. If these uneven deformations or subsidence cannot effectively be controlled, then they will cause damage and even disaster, such as deformation or cracking of buildings.

To sum up, it is difficult to calculate the accurate displacement or subsidence of every point in a body of rock because of the complexity of the factors affecting mine subsidence. Instead, various approximate methods have been used for this calculation. In fact, the movement or subsidence of each point on a level of the overburden can be regarded as a fuzzy event [2,3]. In other words, this displacement or subsidence will take place at a fuzzy probability, and so the theory of fuzzy probability measures can be applied to describe the ground grovements due to deep mining of iron ore by pillarless sublevel caving method.

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2. Fuzzy probability measure models

2.1. Fuzzy probability measure

2.1.1. Fuzzy probability and probabilistic processes

In Ref. [4] (Abraham KANDEL, William J. BYATT. 1980), discussion centered on the algebra of fuzzy sets and on some of the properties of grades of membership in such fuzzy sets. In fact, in ordinary probability theory, probabilities are defined in a given sample space. The collection, in the discrete case, of points in the sample space, where an event *A* occurs, describes the event. Thus, an event is the same as an aggregate of sample points; that is, an event A consists of, or contains, certain points representing an experiment where *A* occurs. Then, by definition, for the discrete case, it follows that the probability of any event *A* is the sum of the probabilities of all sample points in it. We now turn to the question of grades of membership again. In the case wherein we deal with ordinary probability theory, grades of membership in a set can take on only the values unity or zero, corresponding to certain (unity) membership and no (zero) membership. Thus, for example, the grade of membership of an integer in the set of integers is unity, while for non-integer numbers, the grade of membership is zero.

There are, however, concepts for which the notion of either belonging, or not belonging, to a given set make little sense. We give a few examples. Heights are distributed probabilistically. Since, in principle, all heights are possible, we can assign, on the basis of exhaustive experiments, a probability density of heights $P(h)dh$ which gives the probability of having height in the interval $(h,h+dh)$. We want the distribution to be normalized so that $\int_0^\infty P(h)dh = 1$.

But if we now ask the question "what is the probability of the event 'tall' among people whose heights are distributed as above?", then the set of tall people must be defined. We choose to make this definition by incorporating a grade of membership $\mu_T(h) \in [0,1]$ in the set of tall people, with $\lim_{h\to 0}\mu_T(h) \to 0$ and $\lim_{h\to\infty}\mu_T(h) \to 1$. There are other questions which come to mind. We can ask 'what constitutes the set of crowded streets?' We ask "what constitutes a 'suitable' response of a circuit to an input signal?" To answer each of these questions, a suitable mathematics into which is built a notion of grade of membership in a set is needed.

Zadeh [5] has provided a framework such that inexact, or fuzzy, concepts can be discussed rigorously within the confines of an extension of probability theory.

2.1.2. Fuzzy probability theory

First several definitions of fuzzy probability measures are given in brief.

Definition 1. Suppose triplet (Ω,B,ρ) is a probability space, where Ω is a sample space and B is the fuzzy σ-field of Borel sets in Ω (or fuzzy σ-algebra), and ρ is the probability measure over Ω. A_1 and A_2 are two fuzzy sets in the Ω, and μ_{A_1}, μ_{A_2} (μ_{A_1}, μ_{A_2}: $\Omega \to [0,1]$) are two membership functions and μ_{A_1}, μ_{A_2} are Borel measurable.

In Definition 1, $A_1=\{x_1,x_2,\ldots,x_i\}$, $A_2=\{y_1,y_2,\ldots,y_i\}$; where, x_i ($i=1,2,\ldots,n$), y_i are points on the $x0y$ cross section.

Definition 2. If A_1 and A_2 are two fuzzy events in the Ω, then we can define the fuzzy probability measures of A_1 and A_2 as follows:

$$\rho(A_1) = \int_{D_1} \mu_{A_1}(x)d\rho, \quad x \in (0,+\infty), \tag{1}$$

$$\rho(A_2) = \int_{D_2} \mu_{A_2}(y)d\rho, \quad y \in (0,+\infty), \tag{2}$$

Here Eqs. (1) and (2) are Lebesgue–Stieltjes intergrals.

In Definition 2, D_1 is integration area in the x-direction, such as $D_1 \in [0,200]$; and D_2 is integration area in the y-direction, such as $D_2 \in [0,300]$. Where, D_1 ($D_1 = 200$ m) is the mining range in the x-direction), D_2 ($D_2 = 300$ m) is the mining range in the y-direction).

Because μ_{A_1}, μ_{A_2} are Borel measurable, the Lebesgue–Stieltjes integrals exist.

Definition 3. Let A_1 and A_2 be two fuzzy events in the probability space (Ω,B,ρ). A_1 and A_2 are said to be independent if:

$$\rho(A_1A_2) = \rho(A_1)\rho(A_2), \tag{3}$$

where, A_1 A_2 is a product.

Because μ_{A_1}, μ_{A_2} are Borel measurable, the Lebesgue–Stieltjes integrals exist, and because B is a Borel field in the set Ω, it can readily verify that a fuzzy probability measure possesses the following properties:

Property 1. If $A \in B$, then:

$$0 \le \rho(A) \le 1. \tag{4}$$