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Cross-ranking of Decision Making Units in Data Envelopment Analysis

Majid Zerafat Angiz [a], Adli Mustafa [b,*], Mohammad Jalal Kamali [c]

[a] Department of Mathematics, Islamic Azad University, Firouzkooh Branch, Firouzkooh, Iran
[b] School of Mathematical Sciences, Universiti Sains Malaysia, Penang, Malaysia
[c] School of Management, Universiti Sains Malaysia, Penang, Malaysia

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ABSTRACT

Data Envelopment Analysis (DEA) is a mathematical model that evaluates the relative efficiency of Decision Making Units (DMUs) with multiple input and output. In some applications of DEA, ranking of the DMUs are important. For this purpose, a number of approaches have been introduced. Among them is the cross-efficiency method. The method utilizes the result of the cross-efficiency matrix and averages the cross-efficiency scores of each DMU. Ranking is then performed based on the average efficiency scores. In this paper, we proposed a new way of handling the information from the cross-efficiency matrix. Based on the notion that the ranking order is more important than individual efficiency score, the cross-efficiency matrix is converted to a *cross-ranking matrix*. A cross-ranking matrix is basically a cross-efficiency matrix with the efficiency score of each element being replaced with the ranking order of that efficiency score with respect to the other efficiency scores in a column. By so doing, each DMU assume the role of a decision maker and how they voted or ranked the other DMUs are reflected in their respective column of the cross-ranking matrix. These votes are then aggregated using a preference aggregation method to determine the overall ranking of the DMUs. Comparison with an existing cross-efficiency method indicates a relatively better result through usage of the proposed method.

1. Introduction

Data Envelopment Analysis (DEA) is a linear programming technique that was first introduced in Charnes et al. paper [1]. This technique evaluates the relative efficiency of Decision Making Units (DMUs) which contain some nonhomogeneous input and output. In the traditional DEA models like the CCR model [1] and the BCC model [2], efficiency scores of efficient DMUs are 1 and efficiency scores of inefficient DMUs are less than 1. A frequently discussed problem involving the said models is that some of the efficient DMUs cannot be discriminated. A number of approaches have been introduced to overcome this problem and to improve the discrimination power of DEA. Among them are the super-efficiency methods, the multi-criteria DEA methods and the cross-efficiency methods [3].

The super-efficiency method was introduced by Andersen and Petersen [4] and is known as the AP model. In this method, the traditional DEA models are reformulated by excluding the input and output of the DMU that is being ranked from the models. However, this exclusion might cause unfeasibility of the DEA models [4,5]. To overcome this situation, Saati et al. [6] modified the non-radial model presented in Mehrabian et al. [5] and converted it to an input–output orientation model

* Corresponding author. Tel.: +60 04 6533968; fax: +60 04 6570910.
E-mail addresses: mzerafat24@yahoo.com (M. Zerafat Angiz), adli@cs.usm.my (A. Mustafa).

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that caused the LP model to be always feasible. The infeasibility of super-efficiency DEA model in a variable return to scale environment was dealt with by Chen [7] in which both input-oriented and output-oriented super-efficiency DEA models were proposed to fully characterize super-efficiency.

One of the methods which used the multi-criteria decision-making (MCDM) models for ranking DMUs is the method proposed by Li and Reeves [8]. The method introduced a Multi-Criteria Data Envelopment Analysis (MCDEA) model with three objective functions in which the first objective is to obtain the optimal solution of the CCR or the BCC models. The other two objective functions are to minimize the maximum quantity among all deviation variables and to minimize the summation of deviations. Recently, Bal et al. [9] combined the coefficients of variation for the input–output weights to the objective function of the CCR model. However, as noted by Wang and Luo [10], Bal's proposed method has its shortcomings. For instance, since the input and output weights are derived from different dimensions and units, they cannot be simply added. Also, since the model is nonlinear, multiple local optimal solutions may be obtained. Furthermore, Bal et al. [11] converted the MCDEA model proposed by Li and Reeves [8] to a goal programming model in order to discriminate the efficient DMUs.

Cross-efficiency method, pioneered by Sexton et al. [12], evaluates the performance of a DMU by comparing it to the optimal input and output weights of other DMUs. Doyle and Green [13] developed a set of formulations in order to remedy the shortcoming of the Sexton' method which is the non-uniqueness of the factor weights obtained from the DEA models. These models, known as aggressive and benevolent methods, obtain the robust factor weights for use in the construction of the cross-efficiencies method. In the aggressive and benevolent formulations the cross-efficiencies of the other DMUs are minimized and maximized, respectively. Recently, Wang and Chin [14] proposed an alternative cross-efficiency model known as the neutral DEA model to obtain a different set of input and output weights from aggressive and benevolent formulations.

A number of other methods have been integrated with DEA models. Liang et al. [15] integrated game theory and cross-efficiency in order to rank the Decision Making Units in DEA. In the model, DMUs are considered as players. Before this, Sinuany-Stern et al. [16] proposed an approach based on the relationship between DEA and AHP (analytic hierarchy process) to rank DMUs. Recently, Wu [17] utilized the concept of fuzzy for ranking of DMUs in the traditional DEA models. For this purpose, firstly the DMUs are evaluated by the CCR and the cross-efficiency models. Secondly, a fuzzy preference relation is established. Finally, a priority vector of the preference is constructed and is used for ranking DMUs. Zerafat Angiz et al. [18] also utilized the fuzzy concept to completely rank efficient DMUs. On this aspect, this paper integrates the preference aggregation method with DEA to determine the ranking of DMUs.

Preference aggregation problem, in the context of a ranked voting system is a group decision making problem of selecting m alternatives from a set of n alternatives ($n > m$). Hence, each decision maker ranks the alternatives from the most preferred (rank = 1) to the least preferred (rank = n). Obviously, due to different opinions of the decision makers, each alternative may be placed in a different ranking position. Some studies suggest a simple aggregation method by finding the total score of each alternative as the weighted sum of the votes that each alternative received by different decision makers. In this method, the best alternative is the one with the largest total score. The key issue of the preference aggregation is how to determine the weights associated with different ranking positions. Perhaps, Borda–Kendall method [19] is the most commonly used approach for determining the weights due to its computational simplicity. Pioneered by Cook and Kress [20], a number of DEA-based preference aggregation methods have been proposed in recent years. Analysis of these methods and their drawbacks can be found in Llamazares and Peña [21]. Recently, Zerafat Angiz et al. [22,23] also proposed DEA-based methods for handling preference aggregation. The first work was on the use of multi-objective linear programming approach and the second was on the use of fuzzy numbers.

In some DEA applications, pursuing the best ranking is more important than maximizing the individual efficiency score. Wu et al. [24] incorporated this notion into their DEA model by proposing a mixed integer programming model with a secondary goal of ranking order minimization. In this paper, we incorporated this notion by converting the cross-efficiency matrix into a *cross-ranking matrix*. A cross-ranking matrix is basically a cross-efficiency matrix with the efficiency score of each element being replaced with the ranking order of that efficiency score with respect to the other efficiency scores in a column. By so doing, each DMU has now become a decision maker and how they voted or ranked the other DMUs are reflected in their respective column of the cross-ranking matrix. The aggregation of these votes is a preference aggregation problem and a modified Cook and Kress method is used to handle the situation.

The rest of this paper is organized in the following manner. Some mathematical models used in this paper are introduced in Section 2. The proposed method is presented in Section 3. Section 4 illustrates the new method and conclusions are given in Section 4.

2. Background

2.1. CCR model

In mathematical terms, consider a set of n DMUs, in which x_{ij} $(i = 1, 2, \ldots, m)$ and y_{rj} $(r = 1, 2, \ldots, s)$ are input and output of DMU_j $(j = 1, 2, \ldots, n)$. A standard DEA model for assessing DMU_p which is known as the CCR model [1], is formulated in Model (1).