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Numerical solution of Fredholm integral equations of the second kind by using integral mean value theorem II. High dimensional problems

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ABSTRACT

In this work, we generalize the numerical method discussed in [Z. Avazzadeh, M. Heydari, G.B. Loghmani, Numerical solution of Fredholm integral equations of the second kind by using integral mean value theorem, Appl. math. modelling, 35 (2011) 2374–2383] for solving linear and nonlinear Fredholm integral and integro-differential equations of the second kind. The presented method can be used for solving integral equations in high dimensions. In this work, we describe the integral mean value method (IMVM) as the technical algorithm for solving high dimensional integral equations. The main idea in this method is applying the integral mean value theorem. However the mean value theorem is valid for multiple integrals, we apply one dimensional integral mean value theorem directly to fulfill required linearly independent equations. We solve some examples to investigate the applicability and simplicity of the method. The numerical results confirm that the method is efficient and simple.

1. Introduction

Integral equations can describe many different events in science and engineering. Since the most natural phenomena in the real world are related to many factors and parameters, the equations arisen from the modelling are involved to numerous variables. Although there are some different powerful methods to obtain exact and approximate solutions of integral equations, only a few of them can be efficient and applicable to solve the high dimensional problems.

This study is an effort to generalize the new method [1] for solving linear and nonlinear Fredholm integral of the second kind in any dimension and the systems included of any such equations. The methods heretofore available which can solve high dimensional equations are the radial basis functions (RBFs) method [2,3], the spline functions method [20], the block puls functions (BPFs) method [21], the spectral methods such as collocation and Tau method [4–8], Nystrom's method [9,7], transform methods [10], Adomian decomposition method (ADM) [11–13], wavelets methods [14] and many other methods [15–17].

In this paper, we propose the integral mean value method (IMVM) for solving the high dimensional integral equations concerning the first part of this paper [1]. The main idea is applying the mean value theorem for multiple integrals to transform it the system of equations. The obtained system can be solved by Newton's method or other efficient methods [18]. The illustrative examples confirm the validity, efficiency and accuracy of the new method.

This paper is organized as follows: Section 2 briefly reviews the description of the method based on [1] for solving the one dimensional integral equations. In Section 3, we develop the method to solve two dimensional integral equations of the second kind by using the double integral mean value theorem. Section 4 is devoted to generalized the method for solving high

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dimensional integral equations. This section includes the extended formulae to clarify the generalization process. We offer solving system of integral equations using the proposed method in Section 5. We investigate some numerical examples to show the applicability and efficiency of the new method.

2. Description of the integral mean value method (IMVM)

Consider the Fredholm integral equation of second kind as follows

$$u(x) = f(x) + \lambda \int_a^b k(x,t)F(u(t))dt, \quad x,t \in [a,b], \tag{1}$$

where λ is a real number, also F, f and k are given continuous functions, and u is unknown function to be determined. Now we apply the integral mean value theorem for solving the above integral equation.

Theorem 1 (mean value theorem for integrals [19]). *If $\omega(x)$ is continuous in $[a,b]$, then there is a point $c \in [a,b]$, such that*

$$\int_a^b \omega(x)dx = (b-a)\omega(c). \tag{2}$$

Now we describe how (1) can be solved using the integral mean value theorem. By applying (2) for the right hand of (1), since the integral $\int_a^b k(x,t)F(u(t))dt$ depends on x, c will be a function with respect to x and here we write it as $c(x)$. So we can get

$$u(x) = f(x) + \lambda(b-a)k(x,c(x))F(u(c(x))), \tag{3}$$

where $c(x) \in [a,b]$ and $x \in [a,b]$. In practice, to be able to implement our algorithm, we take $c(x)$ as a constant. This assumption results in

$$u(x) = f(x) + \lambda(b-a)k(x,c)F(u(c)), \tag{4}$$

where $c \in [a,b]$. Therefore, finding the value of c and $u(c)$ lead to obtain the solution of integral equation. The way to find c and $u(c)$ is reported from [1] as the following algorithm.

Algorithm

(i) Substitute c into (4) which gives

$$u(c) = f(c) + \lambda(b-a)k(c,c)F(u(c)). \tag{5}$$

(ii) Replace (4) into (1) which gives

$$u(x) = f(x) + \lambda \int_a^b k(x,t)F(f(t) + \lambda(b-a)k(t,c)F(u(c)))dt. \tag{6}$$

(iii) Let c into (6) which lead to

$$u(c) = f(c) + \lambda \int_a^b k(c,t)F(f(t) + \lambda(b-a)k(t,c)F(u(c)))dt. \tag{7}$$

(iv) Solve the obtained equations (5) and (7) simultaneously.

Consecutive substitutions provide the needful linearly independent equations. For solving the above nonlinear system, we can use the various methods [18]. Here, Newton's method is used to solve the obtained system.

3. Solving of two dimensional integral equations via IMVM

Consider the following two dimensional integral equation of the second kind

$$u(x,y) = f(x,y) + \lambda \int_a^b \int_c^d k(x,y,s,t)F(u(s,t))ds\,dt. \tag{8}$$

For solving the above equation, we apply the integral mean value theorem similarly. However the mean value theorem is valid for double integrals, we apply one dimensional integral mean value theorem directly to fulfill required linearly independent equations.

Corollary (mean value theorem for integrals). *If $\omega(x,y)$ is continuous in $[a,b] \times [c,d]$, then there are points $c_1 \in [a,b]$ and $c_2 \in [c,d]$, such that*