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PII: S0893-9659(18)30246-5
DOI: https://doi.org/10.1016/j.aml.2018.07.025
Reference: AML 5597

To appear in: *Applied Mathematics Letters*

Received date : 4 June 2018
Revised date : 14 July 2018
Accepted date : 14 July 2018

Please cite this article as: J. Xu, Z. Liu, S. Shi, Large time behavior of solutions for the attraction–repulsion Keller–Segel system with large initial data, Appl. Math. Lett. (2018), https://doi.org/10.1016/j.aml.2018.07.025

LARGE TIME BEHAVIOR OF SOLUTIONS FOR THE ATTRACTION-REPULSION KELLER-SEGEL SYSTEM WITH LARGE INITIAL DATA

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ABSTRACT. In this paper, we study the following attraction-repulsion Keller-Segel system

$$\begin{cases} u_t = \Delta u - \nabla\cdot(\chi u \nabla v) + \nabla\cdot(\xi u \nabla w), & x\in\Omega,\ t>0,\\ v_t = \Delta v + \alpha u - \beta v, & x\in\Omega,\ t>0,\\ 0 = \Delta w + \gamma u - \delta w, & x\in\Omega,\ t>0,\\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = \frac{\partial w}{\partial \nu} = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x), v(x,0)=v_0(x), & x\in\Omega, \end{cases}$$

in a bounded domain $\Omega\subset\mathbb{R}^2$ with smooth boundary. The boundedness of solutions with arbitrarily large initial data has been proved in the case of $\xi\gamma\ge\chi\alpha$ [8]. Under the additional assumption $\xi\gamma\beta\ge\chi\alpha\delta$, we show that the global classical solution will converge to the unique constant state $(\bar u_0, \frac{\alpha}{\beta}\bar u_0, \frac{\gamma}{\delta}\bar u_0)$ as $t\to+\infty$.

1. INTRODUCTION

Luca et al. [18] proposed the following attraction-repulsion Keller-Segel (ARKS) system

$$\begin{cases} u_t = \Delta u - \nabla\cdot(\chi u \nabla v) + \nabla\cdot(\xi u \nabla w), & x\in\Omega,\ t>0,\\ \tau_1 v_t = \Delta v + \alpha u - \beta v, & x\in\Omega,\ t>0,\\ \tau_2 w_t = \Delta w + \gamma u - \delta w, & x\in\Omega,\ t>0,\\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = \frac{\partial w}{\partial \nu} = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x),\ \tau_1 v(x,0)=\tau_1 v_0(x),\ \tau_2 w(x,0)=\tau_2 w_0(x), & x\in\Omega, \end{cases} \tag{1.1}$$

to describe the aggregation of *Microglia* in the central nervous system, where the parameters $\chi,\xi,\alpha,\beta,\gamma$ and δ are positive, Ω is a bounded domain in $\mathbb{R}^2$ with smooth boundary $\partial\Omega$ and ν denotes the outward normal vector of $\partial\Omega$. $u(x,t)$ stands for the cell density, while $v(x,t)$ and $w(x,t)$ denote the concentrations of the chemoattractant and the chemorepllent, respectively. System (1.1) also was proposed in [19] to model the quorum sensing effect in chemotaxis.

In one dimension, the global existence of classical solutions, non-trivial stationary state, asymptotic behavior and pattern formation of system (1.1) have been studied in [7, 16, 17]. In two dimensions, when $\tau_1=\tau_2=1$, it was shown in [21] that global classical solutions exist for large data if $\beta=\delta$ and for small data if $\beta\ne\delta$ when $\xi\gamma\ge\chi\alpha$ (i.e. the repulsion dominates over or cancels the attraction). Subsequently, the global existence of large-data solutions was extended to the case $\beta\ne\delta$ in [6, 15]. Moreover, it was shown that the global classical solution will exponentially converge to the unique non-trivial constant state $(\bar u_0, \frac{\alpha}{\beta}\bar u_0, \frac{\gamma}{\delta}\bar u_0)$ with $\bar u_0 = \frac{1}{|\Omega|}\int_\Omega u_0$ in [12, 13] if the cell mass is small or if the cell mass is large and $\frac{\xi\gamma}{\chi\alpha}\ge\max\{\frac{\beta}{\delta},\frac{\delta}{\beta}\}$ [10]. If $\tau_1=\tau_2=0$, it has been proven that system (1.1) has a unique classical solution with uniform-in-time bound if $\xi\gamma\ge\chi\alpha$ (i.e. the repulsion dominates over or cancels the attraction) in higher dimensions ($n\ge2$) [21]. If $\xi\gamma<\chi\alpha$ (i.e. the attraction dominates over the repulsion), it was shown that the solution of system (1.1) exists globally for the small initial mass and will blow up in finite time if the initial mass is large in two dimensions [4, 11].

2000 *Mathematics Subject Classification.* 35A01, 35B40, 35B44, 35K57, 35Q92, 92C17.
Key words and phrases. Chemotaxis, attraction-repulsion, global stability.
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