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ON A RIEMANN-HILBERT PROBLEM FOR THE FOKAS-LENELLS EQUATION

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Abstract. The solution of the initial value problem (IVP) for the Fokas-Lenells equation (FLE) was constructed in terms of the solution $M(x,t,k)$ of a 2×2 matrix Riemann-Hilbert problem (RHP) as $k\to\infty$, and the one-soliton solution of the FLE was derived based on this Riemann-Hilbert problem, in Nonlinearity 22(2009), [1]. However, in fact, the derivative with respect to x of the solution of the FLE ($u_x(x,t)$) was recovered from the RHP as $k\to\infty$. In this paper, we construct the solution of the FLE in terms of the RHP as $k\to 0$, because the Lax pair of the FLE contains the negative order of the spectral variable k. We show that the one-soliton solution of the FLE obtained in this paper is the same as [1], but avoiding a complex integral.

1. Introduction

The Fokas-Lenells equation (FLE)

$$u_{tx}+\alpha\beta^2u-2i\alpha\beta u_x-\alpha u_{xx}+\sigma i\alpha\beta^2|u|^2u_x=0,\quad \sigma=\pm 1, \tag{1.1}$$

where $\alpha>0$ and β are real constant, $u(x,t)$ is a complex-valued function, while the subscripts t and x denote the partial derivativations. It is a completely integrable nonlinear partial differential equation (here "integrable" we mean it admits a Lax pair) which has been derived as an integrable generalization of the nonlinear Schrödinger equation (NLSE) using bi-Hamiltonian methods [2]. In the context of nonlinear optics, the FLE models the propagation of nonlinear light pulses

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