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A Trefftz/MFS mixed-type method to solve the Cauchy problem of the Laplace equation

Chein-Shan Liu[a,b], Fajie Wang[c,**], Yan Gu[d,*]

[a]Center for Numerical Simulation Software in Engineering and Sciences,
College of Mechanics and Materials, Hohai University, Nanjing, Jiangsu 210098, China

[b]Center of Excellence for Ocean Engineering, Department of Mechanical and Mechatronic Engineering,
National Taiwan Ocean University, Keelung 202-24, Taiwan

[c]School of Electromechanic Engineering, Qingdao University, Qingdao 266071, China

[d]College of Mathematics, Qingdao University, Qingdao 266071, China

[*,**]Corresponding authors, E-mails: guyan1913@163.com, wfj1218@126.com

Abstract

By using the multiple-scale Trefftz method (MSTM) to solve the Cauchy problem of the Laplace equation in an arbitrary bounded domain, we may lose the accuracy several orders when the noise being imposed on the specified Cauchy data is quite large. In addition to the linear equations obtained from the MSTM, the fundamental solutions play as the test functions being inserted into a derived boundary integral equation. Therefore, after merely supplementing a few linear equations in the mixed-type method (MTM), which is a well organized combination of the Trefftz method and the method of fundamental solutions (MFS), we can improve the ill-conditioned behavior of the linear equations system and hence increase the accuracy of the solution for the Cauchy problem significantly, as explored by two numerical examples.

Keywords: Laplace equation, Cauchy problem, Trefftz method, MFS test functions, Mixed-type method

1. Introduction

For solving the Laplace equation in a two-dimensional bounded and simply connected domain, it is well-known that the set

$$\{1, r^k \cos k\theta, r^k \sin k\theta,\ k = 1, \ldots, \infty\} \tag{1}$$

forms the T-complete functions, and the numerical solution in terms of these bases is named the Trefftz method. The Trefftz method is a truly meshless boundary type method, which leaves the unknown expansion coefficients determined merely by the specified boundary conditions.

However, for the Trefftz method the resultant linear equations system to determine the expansion coefficients is notoriously ill-conditioned, and the accuracy drops down quickly when the order of the Trefftz bases is increased to over a critical value. A bad choice of the order of the Trefftz bases and the collocation points may cause the numerical solution unstable. In the present paper we develop a mixed-type method from the Trefftz method and the method of fundamental solutions (MFS); however, we employ the fundamental solutions as the test functions in a derived