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Balancing domain decomposition by constraints associated with subobjects

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Abstract

A simple variant of the BDDC preconditioner in which constraints are imposed on a selected set of subobjects (subdomain subedges, subfaces and vertices between pairs of subedges) is presented. We are able to show that the condition number of the preconditioner is bounded by $C\big(1+\log(L/h)\big)^2$, where C is a constant, and h and L are the characteristic sizes of the mesh and the subobjects, respectively. As L can be chosen almost freely, the condition number can theoretically be as small as $O(1)$. We will discuss the pros and cons of the preconditioner and its application to heterogeneous problems. Numerical results on supercomputers are provided.

Keywords: BDDC, FETI-DP, optimal preconditioner, parallel solver, heterogeneous problems
2010 MSC: 65N55, 65N22, 65F08

1. Introduction

It is well-known in the literature [12, 13, 14, 16, 8, 7] that the condition number of the BDDC method and the closely related FETI-DP method is bounded by $\kappa \leq C\big(1+\log(H/h)\big)^2$, where C is a constant, and H and h are the characteristic sizes of the subdomain and the mesh, resp. Consequently, when the mesh and its partition are fixed, users can only vary the number of constraints to adjust the convergence rate.

Traditionally, these constraints are values at subdomain corners, or average values on subdomain edges or faces. In the perturbed formulation of BDDC [6, 5], any combination of vertex, edge or face constraints can be used. Normally, these combinations provide enough options for users to choose their desired convergence rate. However, for difficult problems, such as the ones with heterogeneous diffusion coefficients, see, e.g., [1], the convergence of the BDDC method [9] can be unsatisfied even if all possible constraints are used.

In this work, we propose to impose constraints on coarse objects (subdomain subedges and subfaces, and vertices between subedges) that are adapted to the local mesh size and to the user's desired rate of convergence. We refer to this new variant as BDDC-SO. If L is the characteristic size of the objects, we can prove that the conditioned number κ of the BDDC-SO preconditioned operator is bounded by

$$\kappa \leq C\big(1+\log(L/h)\big)^2.$$

As L can be as small as h, there is no restriction in the rate of convergence of BDDC-SO. However, the trade-off is having to solve larger coarse problems. Therefore, the BDDC-SO can be seen as a standalone method for small to medium problems (tested up to 260 millions unknowns on 8K processors), or as a proven, simple approach to improve the rate of convergence. In addition, BDDC-SO is perfectly robust w.r.t. the variation of the diffusion coefficient when coarse objects are chosen such that the coefficient associated with each object is constant or varies mildly. BDDC-SO also has great potentials to be extended to multi-level.

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