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Wei Zheng, Jitsuro Sugie

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Uniform global asymptotic stability of time-varying Lotka-Volterra predator-prey systems

Wei Zheng[a], Jitsuro Sugie[*,a]

[a]*Department of Mathematics, Shimane University, Matsue 690-8504, Japan*

Abstract

The model to be dealt in this paper is

$$N' = (a + ch(t) - dh(t)N - bP)N,$$
$$P' = (-c + dN)P.$$

Here, h is a nonnegative and locally integrable function. This model is a predator-prey system of Lotka-Volterra type with variable coefficients and it has a single interior equilibrium $(c/d, a/b)$. Sufficient conditions are given for the interior equilibrium to be uniformly globally asymptotically stable. One of them is described by using a certain uniform divergence condition on h. Our result is proved by examining in details the behaviour of all solutions of a planar system equivalent to this model.

Key words: Uniform global asymptotic stability; Lotka-Volterra predator-prey model; Uniform divergence; Growth condition; Time-varying system
2010 MSC: 34C60; 34D05; 34D23; 92D25

1. Introduction

The population of species in ecological models is one of the most important issues in biomathematics. To understand the dynamics of species populations, mathematical models are often used. For example, the classical Lotka-Volterra predator-prey system was first raised up to describe the population of sharks and fish in the Adriatic Sea. Afterwards, this model was progressively improved so that it can be used not only for ecology but also for analysis of physical phenomena and economical theory. In the process, it was pointed out that this model is structurally unstable and there is a gap with natural phenomena. For this reason, many researchers have paid various efforts to find more reasonable models that describe nature. (for example, see [1, 2, 3, 4, 5, 8, 9, 10, 11] and the references cited therein). Among them, the idea that the seasonal change should be emphasised has arisen. Because the environment, the habitat state and other related factors can change over season, this idea that alternation of season is one of the basic factors that can affect population ecology will be reasonable.

In this paper, taking into account that the prey is more susceptible to the seasonal change than the predator, we deal with the time-varying system

$$\begin{aligned} N' &= \alpha(t)N - \beta(t)N^2 - bNP, \\ P' &= -cP + dNP, \end{aligned} \tag{E}$$

where $' = d/dt$, the letters N and P represent the density of prey and predator population, respectively. Functions α and β are the intrinsic growth rate of prey and the density limiting rate due to the intraspecific competition, respectively. Parameters b, c, and d are the predation rate of predator on prey, the death rate of predator, and the rate at which predator increases by consuming prey, respectively. Since population can not be negative, it is natural to consider model (E) in the first quadrant $Q \stackrel{\text{def}}{=} \{(N, P) : N \geq 0 \text{ and } P \geq 0\}$.

*Corresponding author
Email addresses: vividloky@yahoo.co.jp (Wei Zheng), jsugie@riko.shimane-u.ac.jp (Jitsuro Sugie)