Accepted Manuscript

Boundary value problem for linear and nonlinear fractional differential equations

Tengteng Ma, Yu Tian, Qiang Huo, Yong Zhang

PII: S0893-9659(18)30187-3
DOI: https://doi.org/10.1016/j.aml.2018.06.010
Reference: AML 5544

To appear in: *Applied Mathematics Letters*

Received date : 16 March 2018
Revised date : 9 June 2018
Accepted date : 9 June 2018

Please cite this article as: T. Ma, Y. Tian, Q. Huo, Y. Zhang, Boundary value problem for linear and nonlinear fractional differential equations, Appl. Math. Lett. (2018), https://doi.org/10.1016/j.aml.2018.06.010

Boundary value problem for linear and nonlinear fractional differential equations *

Tengteng Ma[a,b],† Yu Tian[b], Qiang Huo[b], Yong Zhang[a]

a. School of Electronic Engineering, Beijing University of Posts and Telecommunications,

Beijing 100876, P.R. China

b. School of Science, Beijing University of Posts and Telecommunications,

Beijing 100876, P.R. China

Abstract: In this paper, we are interested in the boundary conditions $u(0) = au(1)$, $u'(0) = bu'(1)$ for linear fractional differential equation ${}_0^cD_t^\alpha u(t) + \lambda u(t) = 0$. Via Laplace transform and inverse Laplace transform, we obtain eigenvalues and eigenfunctions. Furthermore, we study the same boundary conditions for nonlinear fractional differential equation ${}_0^cD_t^\alpha u(t) + f(t, u(t)) = 0$. Combining the obtained eigenvalues and eigenfunctions and the improved Leray-Schauder degree, we prove that there exists at least one nontrivial solution for nonlinear boundary value problem.

Keywords: Fractional differential equation; Eigenvalue; Green function; Improved Leray-Schauder degree; Nontrivial solution.

1 Introduction

In recent years, fractional differential equations have attracted many attentions since they can be applied in many fields of science and engineering, such as, viscoelasticity, neurons, BP neural network, electrochemistry, control, porous media, etc, see [3, 8, 12–14, 16]. For background and applications of the theory of fractional differential equations, we refer the readers to the monographs [5, 6, 9, 15].

Recently, fractional differential equations have been studied by many researchers [2,7,17–24]. For instance, in 2005, Bai [23] studied nonlinear fractional differential equation with Dirichlet boundary condition. The author used fixed-point theorem on cone to get some existence and multiplicity results of positive solutions. In 2011, Zhao [22] studied nonlinear fractional differential equation with the mixed set of Neumann and Dirichlet boundary conditions. The author studied the nonexistence and existence of at least one or two positive solutions for the boundary value problem. With boundary conditions $u'(0) = 0$ and $u(1) = 0$, [19]

*This work was supported by Research Innovation Fund for College Students of Beijing University of Posts and Telecommunications [grant number 1707003].

†Corresponding author: buptteng@foxmail.com