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On Convergence of EVHSS Iteration Method for Solving Generalized Saddle-Point Linear Systems *

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Abstract

For the generalized saddle-point linear system, we prove the unconditional convergence of the efficient variant of the HSS (EVHSS) iteration method introduced by Zhang (J.-L. Zhang, Numer. Linear Algebra Appl. (2018), e2166: 1-14.).

Keywords: generalized saddle-point linear system, matrix splitting iteration, convergence.

AMS(MOS) Subject Classifications: 65F10, 65F15; CR: G1.3.

1 Introduction

Let us consider generalized saddle-point linear systems of the form

$$Ax \equiv \begin{pmatrix} B & E \\ -E^* & C \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix} \equiv b, \tag{1.1}$$

where $B \in \mathbb{C}^{p\times p}$ and $C \in \mathbb{C}^{q\times q}$ are Hermitian positive semidefinite matrices, and $E \in \mathbb{C}^{p\times q}$ is a rectangular matrix, satisfying

$$\mathrm{null}(B) \cap \mathrm{null}(E^*) = \{0\} \quad \text{and} \quad \mathrm{null}(C) \cap \mathrm{null}(E) = \{0\}.$$

Then the generalized saddle-point matrix $A \in \mathbb{C}^{n\times n}$, with $n = p+q$, is nonsingular, and it follows that the generalized saddle-point linear system (1.1) has a unique solution for any right-hand side $b = (f^*, g^*)^*$, with $f \in \mathbb{C}^p$ and $g \in \mathbb{C}^q$; see, e.g., [3]. Here, $(\cdot)^*$ and $\mathrm{null}(\cdot)$ indicate the conjugate transpose and the null space of the corresponding matrix, respectively.

In accordance with the *Hermitian and skew-Hermitian splitting* (**HSS**) preconditioner [12, 13, 15, 1] and in the spirit of its preconditioning, modification, generalization, relaxation and regularization strategies [13, 17, 20, 7, 23, 24, 25, 30], Zhang further proposed in [28] an *efficient variant*, briefly denoted as EVHSS[1] preconditioner, for the generalized saddle-point matrix $A \in \mathbb{C}^{n\times n}$ as follows:

$$M_{\mathrm{EVHSS}}(\alpha) = \frac{1}{\alpha} \begin{pmatrix} B & E \\ -E^* & \alpha I \end{pmatrix} \begin{pmatrix} \alpha I & 0 \\ 0 & \alpha I + C \end{pmatrix}, \tag{1.2}$$

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[1]EVHSS is an abbreviation of the terminology *'efficient variant of HSS'*