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Scott-Blair models with time-varying viscosity

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Abstract

In a recent paper, Zhou et al. [12] studied the time-dependent properties of Glass Fiber Reinforced Polymers composites by employing a new rheological model with a time-dependent viscosity coefficient. This rheological model is essentially based on a generalized Scott-Blair body with a time-dependent viscosity coefficient. Motivated by this study, in this note we suggest a different generalization of the Scott-Blair model based on the application of Caputo-type fractional derivatives of a function with respect to another function. This new mathematical approach can be useful in viscoelasticity and diffusion processes in order to model systems with time-dependent features. In this paper we also provide the general solution of the creep experiment for our improved Scott-Blair model together with some explicit examples and illuminating plots.

Keywords: Scott-Blair model, fractional derivative, Mittag-Leffler function, creep experiment, viscoelasticity, polymer rheology

1. Introduction

In the 50's, the British chemist George William Scott-Blair introduced the first linear viscoelasticty model with physical properties intermediate between the elastic Hooke model and the fluid Newton model. The mathematical implementation of Scott-Blair's ideal system was performed by means of the notion of derivative of non-integer order, see *e.g.* [1] for an historical perspective.

Unfortunately, as reported by Stiassnie in [2], after several experimental contributions in the emerging new field of viscoelasticity, ultimately Scott-Blair gave up on studying the implications of non-integer order calculus in rheology as he could not find a satisfactory mathematical definition of "fractional differential".

Since the time of Scott-Blair, the theory of fractional calculus [3–7], that is the mathematical discipline dealing with integrals and derivatives of non integer order, has been refined and represents one of the most important languages for the modern formulation of linear viscoelasticity [5, 6]. Besides, it is worth noting that fractional calculus plays a central role also in many other fields of science, see *e.g.* [8–11] and references therein.

In a recent paper, Zhou et al. [12] investigated the time-dependent properties of Glass Fiber Reinforced Polymers (GFRP) composites by means of a generalized Scott-Blair model with time-varying viscosity. Furthermore, it is worth remarking that viscoelastic models featuring a time-varying viscosity have also been analyzed in [13] by Pandey and Holm.

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