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Modeling of atmospheric temperature fluctuations by translations of oscillatory random processes with application to spacecraft atmospheric re-entry

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ABSTRACT

The presence of random fluctuations of air temperature within the Earth's atmosphere is a well-documented phenomenon. During the past seventy years there have been numerous experimental efforts to accurately measure air temperature as a function of altitude and, through careful data analysis, provide statistics describing these fluctuations and the associated fluctuations in temperature gradients. In addition, several researchers suggest the presence of atmospheric layers or "sheets" where the statistics describing fluctuations in air temperature can vary significantly from layer to layer. Herein, we propose a model to represent fluctuations of air temperature within a layered atmosphere. The model is a special type of inhomogeneous non-Gaussian differentiable random process and can be calibrated to available data on the marginal statistics and spectral content of the fluctuating temperature field, as well as the associated first derivative of the process representing fluctuations in temperature gradients. Properties of the proposed model are presented, and statistical realizations of the fluctuating temperature field and its gradient are computed and presented for illustration. The random vibration response of a spacecraft falling to Earth through these fluctuating conditions is then considered to demonstrate the usefulness of the proposed model.

1. Introduction

Random fluctuations of air temperature in the Earth's atmosphere are a ubiquitous phenomenon. The quantification of these fluctuations is of interest for a variety of reasons including: (i) the development of general theories of atmospheric turbulence [1,2]; (ii) quantifying the removal of heat and water vapor from the ground into the surrounding atmosphere [3,4]; (iii) improving the accuracy of radar measurements [5,6]; and (iv) achieving a better understanding of the diffusion of electromagnetic and acoustic waves [7]. Through both laboratory and field experiments, several researchers have measured small-scale random fluctuations in air temperatures and temperature gradients, providing some statistics and estimates of marginal distributions [8,3,9,10]. Others have demonstrated evidence of larger-scale atmospheric layers where fluctuations in temperatures and temperature gradients behave very differently [5].

Herein, we are concerned with quantifying the dynamic response of a spacecraft undergoing ballistic re-entry. Typically, this is achieved through standard trajectory analyses; see, for example, the books by Martin [11] or Regan and Anandakrishnan [12]. However, because a completely deterministic model for the atmosphere is assumed, this approach is limited to quasi-static response of the vehicle. Herein, we include a model for temperature fluctuations within the Earth's atmosphere as input to a trajectory analysis, and assess the resulting random vibration response of the spacecraft. The approach taken here appears to be the first attempt to understand the effect that fluctuations in atmospheric conditions might have on a spacecraft undergoing ballistic re-entry.

The proposed model for temperature fluctuations is a particular type of non-Gaussian random process called a translation process, defined by a nonlinear transformation of a Gaussian process [13, Section 3.1.1]. By careful selection of the properties of the Gaussian process and the functional form of the transformation, it is possible to calibrate the translation process to available data on small-scale fluctuations of both temperature and temperature gradients, including the variance, coefficient of kurtosis, and spectral content. The effects of the larger-scale atmospheric layers noted by Dalaudier et al. [5] can also be captured by the proposed model. While there have been numerous applications of translation processes in structural engineering and mechanics, including the modeling of material properties [14,15], aerodynamic loads on

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structures [16,17], ocean wave heights [18], and ground motion due to seismic events [19], the work proposed herein appears to be the first attempt to use translation processes to represent atmospheric temperature fluctuations.

The organization of the paper is as follows. Section 2 provides a discussion of some of the properties of translation processes, including the properties of the first derivative of a mean-square differentiable translation process, which is used to represent fluctuations in temperature gradients. The application of the translation process and its first derivative to represent fluctuations in atmospheric temperature and temperature gradients is presented in Section 3. Initially, we model fluctuations within a single atmospheric layer by a suitable transformation of a homogeneous Gaussian process. The proposed model is then generalized to represent temperature fluctuations within a layered atmosphere by considering transformations of a particular type of inhomogeneous Gaussian process called an oscillatory process [20]. This approach allows for the changing variance and spectral content that is typical of different layers in the atmosphere [8]. In Section 4, we consider the atmospheric re-entry of a spacecraft as it falls to Earth through the fluctuating atmosphere, and assess the resulting random vibration response. For completeness, a brief discussion on oscillatory processes is presented in the Appendix.

2. Translation processes

Let $z \geq 0$ denote a spatial coordinate, *e.g.*, the geometric altitude above the surface of the Earth, and define $G(z)$ to be a real-valued, mean-square differentiable, homogeneous Gaussian random process with zero mean, unit variance, and covariance function $r_G(\eta) = \mathrm{E}[G(z)\,G(z+\eta)]$. A translation process is defined to be a monotonic increasing, z-invariant transformation of G of the following form [13, Section 3.1.1]

$$Y(z) = F_Y^{-1} \circ \Phi(G(z)) = h(G(z)), \tag{1}$$

where F_Y is an arbitrary cumulative distribution function (CDF), and Φ denotes the CDF of the standard Gaussian random variable. Herein, we assume F_Y is absolutely continuous, that is, we assume there exists an integrable function f_Y such that $f_Y(y) = \mathrm{d}F_Y(y)/\mathrm{d}y$ is a probability density function (PDF). It can be shown that Y is a homogeneous process with marginal CDF F_Y, marginal PDF f_Y, and one-sided spectral density function

$$g_Y(\kappa) = 4\int_0^\infty r_Y(\eta)\,\cos(2\pi\kappa\,\eta)\,\mathrm{d}\eta, \tag{2}$$

where $\kappa = 1/\lambda \geq 0$ is a spatial frequency or wavenumber corresponding to a wave with wavelength λ, and

$$\begin{aligned} r_Y(\eta) &= \mathrm{E}[Y(z)\,Y(z+\eta)] \\ &= \int_{-\infty}^\infty \mathrm{d}u_1 \int_{-\infty}^\infty h(u_1)\,h(u_2)\,\phi(u_1,u_2;r_G(\eta))\,\mathrm{d}u_2, \end{aligned} \tag{3}$$

is the correlation function of Y with $\phi(u_1,u_2;r_G(\eta))$ representing the joint PDF of dependent Gaussian random variables $G(z)$ and $G(z+\eta)$ with correlation coefficient $r_G(\eta)$.

The first derivative of $Y(z)$ is

$$Y'(z) = \frac{\mathrm{d}Y(z)}{\mathrm{d}z} = \frac{\mathrm{d}h}{\mathrm{d}G}\,\frac{\mathrm{d}G}{\mathrm{d}z} = h'(G(z))\cdot G'(z), \quad z \geq 0, \tag{4}$$

where the existence of h' is guaranteed since F_Y is absolutely continuous. We can show that: (i) $G'(z) = \mathrm{d}G(z)/\mathrm{d}z$ is a homogeneous Gaussian process with zero mean and variance $\sigma_{G'}^2 = r_{G'}(0)$, where $r_{G'}(\eta) = \mathrm{E}[G'(z)\,G'(z+\eta)] = -r_G''(\eta)$ is the correlation function of $G'(z)$; (ii) $G(z)$ and $G'(z)$ are independent random variables for each z; (iii) $Y'(z)$ is a homogeneous process with zero mean; and (iv) unless h is a constant, the derivative of a translation process is not a translation process.

The marginal distribution of $Y'(z)$ defined by Eq. (4) is obtained by integration of the joint PDF of random vector $(G(z), G'(z))^T$ for

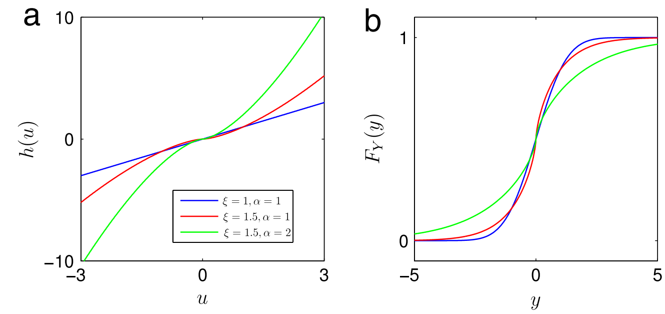

Fig. 1. Transformation function h, panel (a), and marginal CDF of translation process F_Y, panel (b), for various parameters ξ and α.

fixed z over a suitable domain, *i.e.*,

$$F_{Y'}(a) = \Pr\left(Y'(z) \leq a\right) = \iint_{v\,h'(u)\leq a} \phi(u)\,\phi\left(\frac{v}{\sigma_{G'}}\right)\mathrm{d}u\,\mathrm{d}v, \tag{5}$$

where the integrand is a separable function of u and v since G and G' are independent. By Eq. (4) and properties of Gaussian random variables, the moments of order $p \geq 1$ of $Y'(z)$ are

$$\begin{aligned} m_{p,Y'} &= \mathrm{E}[Y'(z)^p] = \mathrm{E}[h'(G(z))^p]\,\mathrm{E}[G'(z)^p] \\ &= \begin{cases} 0, & p = 1,3,\ldots \\ 1\cdot 3\cdots(p-1)\,\sigma_{G'}^p\,\mathrm{E}[h'(G(z))^p], & p = 2,4,\ldots. \end{cases} \end{aligned} \tag{6}$$

In particular, Y' has zero mean, zero skewness, and a coefficient of kurtosis equal to $3\,\mathrm{E}[h'(G(z))^4]/(\mathrm{E}[h'(G(z))^2])^2$. The correlation, covariance, and spectral density functions of $Y'(z)$ are $r_{Y'}(\eta) = c_{Y'}(\eta) = \mathrm{E}[Y'(z)\,Y'(z+\eta)] = -r_Y''(\eta)$ and $g_{Y'}(\kappa) = (2\pi\kappa)^2\,g_Y(\kappa)$, respectively.

We next consider a particular translation process that will prove useful for modeling temperature fluctuations in Section 3. Define

$$Y(z) = \alpha|G(z)|^\xi\,\mathrm{sgn}(G(z)) = h(G(z)), \quad z \geq 0, \tag{7}$$

where $\alpha, \xi > 0$ are parameters, and $\mathrm{sgn}(u) = -1, 0$, or 1 for $u < 0$, $u = 0$, or $u > 0$, respectively. Process $Y(z)$ is a translation process since it is a monotonic increasing, z-invariant transformation of Gaussian process G; Y is Gaussian if, and only if, $\xi = 1$. The marginal CDF of $Y(z)$ is

$$\begin{aligned} F_Y(y) &= \int_{\alpha|u|^\xi\,\mathrm{sgn}(u)\leq y} \phi(u)\,\mathrm{d}u = \int_{-\infty}^{|y/\alpha|^{1/\xi}\,\mathrm{sgn}(y)} \phi(u)\,\mathrm{d}u \\ &= \Phi\left(\left|\frac{y}{\alpha}\right|^{1/\xi}\,\mathrm{sgn}(y)\right). \end{aligned} \tag{8}$$

Transformation h defined by Eq. (7) and the corresponding marginal CDF of Y defined by Eq. (8) are illustrated by Fig. 1 for different values of ξ and α. The distribution is symmetric about $y = 0$ for all ξ and α and, as ξ increases, the rate of decay of the tails of the distribution decreases. Further, $Y(z)$ has zero mean and moments

$$m_{p,Y} = \begin{cases} 0, & p = 1,3,\ldots \\ \dfrac{\alpha^p\,2^{\xi p/2}}{\sqrt{\pi}}\,\Gamma\left(\dfrac{\xi p+1}{2}\right), & p = 2,4,\ldots \end{cases} \tag{9}$$

where $\Gamma(\cdot)$ denotes the Gamma function [21, Chapter 6]. For example, the variance and coefficient of kurtosis of $Y(z)$ are $(\alpha^2\,2^\xi/\sqrt{\pi})\,\Gamma(\xi+1/2)$ and $\sqrt{\pi}\,\Gamma(2\xi+1/2)/(\Gamma(\xi+1/2))^2$, respectively.

By Eqs. (4) and (7),

$$Y'(z) = \xi\,\alpha|G(z)|^{\xi-1}G'(z), \quad z \geq 0, \tag{10}$$