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Existence and uniqueness of a Lotka-Volterra reaction-diffusion model with advection term☆

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Abstract

In this paper, we investigate a reaction-diffusion two-competition species model with advection term under the Dirichlet boundary condition. By employing implicit function theorem, the principal spectral theory and other important formulas, we obtain the existence and uniqueness of spatial nonhomogeneous solutions under some given parameters conditions.

Keywords: Competition species; reaction-diffusion-advection; spatial nonhomogeneous solutions; implicit function theorem.

1. Introduction

During the past several tens of years, the dynamics models in the form of reaction-diffusion systems have been investigated extensively for homogeneous reaction-diffusion model, one can refer to [2, 4, 5, 6, 10, 11]. It is well-known that the positive steady state solutions of the systems with the homogeneous Dirichlet boundary value conditions is always spatially nonhomogeneous. For example, Ma and Guo[8] considered the existence and multiplicity of the positive steady state solutions of the following two population reaction-diffusion model:

$$\begin{cases} \dfrac{\partial u(t,x)}{\partial t} = d\Delta u(t,x) + lu(t,x) - u^2(t,x) - B\phi(u(t,x))v(t,x), \quad x \in \Omega,\ t \geq 0, \\ \dfrac{\partial v(t,x)}{\partial t} = d\Delta v(t,x) + mv(t,x) - Dv^2(t,x) + \phi(u(t,x))v(t,x), \quad x \in \Omega,\ t \geq 0, \\ u(t,x) = v(t,x) = 0, \quad x \in \partial\Omega, \quad t \geq 0, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 1$, is a bounded domain with a smooth boundary. Besides B and D are strictly positive, Δ denotes the Laplacian operator, d is the diffusion coefficient. In biology, u and v can be interpreted as the densities of prey and predator populations, respectively, and D denotes the self-limitation constants. l and m denote the intrinsic growth rate of the prey and the predator, respectively. In the presence of predators and the diffusion effect, there is a hunting term $B\phi(u)$

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