Accepted Manuscript

Dwell time for local stability of switched affine systems with application to non-spiking neuron models

Oleg Makarenkov, Anthony Phung

PII: S0893-9659(18)30203-9
DOI: https://doi.org/10.1016/j.aml.2018.06.026
Reference: AML 5560

To appear in: *Applied Mathematics Letters*

Received date : 19 April 2018
Revised date : 19 June 2018
Accepted date : 19 June 2018

Please cite this article as: O. Makarenkov, A. Phung, Dwell time for local stability of switched affine systems with application to non-spiking neuron models, Appl. Math. Lett. (2018), https://doi.org/10.1016/j.aml.2018.06.026

Dwell time for local stability of switched affine systems with application to non-spiking neuron models

Oleg Makarenkov[a,*], Anthony Phung[a]

[a]*Department of Mathematical Sciences, University of Texas at Dallas, 75080, TX, Richardson, USA*

Abstract

For switched systems that switch between distinct globally stable equilibria, we offer closed-form formulas that lock oscillations in the required neighborhood of the equilibria. Motivated by non-spiking neuron models, the main focus of the paper is on the case of planar switched affine systems, where we use properties of nested cylinders coming from quadratic Lyapunov functions. In particular, for the first time ever, we use the dwell-time concept in order to give an explicit condition for non-spiking of linear neuron models with periodically switching current.

Keywords: Switched system, dwell-time, trapping region, multiple equlibria, planar switched affine systems, non-spiking, subshreshold oscillations, linear neuron model
2000 MSC: 93C30, 34D23, 92C20

1. Introduction

Dwell time is the lower bound on the time between successive discontinuities (switchings) of the piecewise constant function $u(t)$, which ensures that the corresponding switched (affine in our case) system

$$\dot{x} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} x + B_{u(t)}, \qquad x \in \mathbb{R}^2, \tag{1}$$

where $a, b, c, d \in \mathbb{R}$ and B_u is a u-dependent vector of $\mathbb{R}^2$, exhibits a required type of stability, under the assumption that each of the subsystems of (1) possess a unique globally asymptotically stable equilibrium x_u. Let V_u be some Lyapunov function of subsystem (1) corresponding to $u(t) = x_u$ and let N_u^k be the neighborhood of x_u given by

$$N_u^k = \{x : V_u(x) \le k\}. \tag{2}$$

Extending the pioneering result by Alpcan-Basar [1] (see also Liberzon [7, §3.2.1]), the recent paper [4] by Dorothy and Chung gives an important formula for the dwell time τ_d which ensures that any solution of (1) with the initial condition $x(t_0) \in N_{u(t_0)}^k$ satisfies

$$x(t_i) \in N_{u(t_i)}^k, \quad i \in \mathbb{N}, \tag{3}$$

as long as the successive discontinuities $t_1, t_2, ...$ of the control signal $u(t)$ verify $t_{i+1} - t_i \ge \tau_d$, $i \in \mathbb{N}$. At the same time, the results of [4] are formulated in general abstract settings and certain work is required to apply those results to particular problems. In the present paper we follow the strategy of [4] when addressing planar switched affine systems, but carry out an independent proof that allows us to get closed-form formulas for the dwell-time τ_d (i.e. formulas in terms of just coefficients of the affine subsystems).

Relevant significant results have been recently obtained in Xu et al [11] for quasi-linear switched systems (1), but the dwell-time formula [11] is not fully explicit, as it involves the constant of the rate of decay of the matrix exponent of the homogeneous part of subsystems (1).

*Corresponding author
Email address: makarenkov@utdallas.edu (Oleg Makarenkov)