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Preconditioned symmetric block triangular splitting iteration method for a class of complex symmetric linear systems

Jianhua Zhang[a,*], Zewen Wang[a], Jing Zhao[a]

[a]*School of Science, East China University of Technology, Nanchang, Jiangxi, 330013, China*

Abstract

In this paper, a preconditioned symmetric block triangular splitting (PSBTS) iteration method is proposed for solving a class of block two-by-two real linear systems which arise from complex symmetric linear systems. The eigenvalue distributions and the convergence properties of the PSBTS iteration method are established, and the optimal values of the relaxation parameter and the corresponding spectral radius of the PSBTS iteration matrix are obtained, respectively. It is remarkable that the spectral radius of the PSBTS iteration method is smaller than that of the SBTS iteration method under suitable conditions. Finally, two numerical examples are tested for illustrating effectiveness and robustness of the PSBTS iteration method.

Keywords: Block two-by-two matrix, Complex symmetric linear system, Splitting iteration method

1. Introduction

Consider a block two-by-two linear system of the following form

$$\mathcal{A}\mathbf{x} = \begin{bmatrix} W & -T \\ T & W \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix} \equiv \widetilde{b}, \tag{1}$$

which is a real equivalent formulation of the following complex symmetric system of linear equations

$$(W + iT)(x + iy) = p + iq, x, y, p, q \in \mathbb{R}^n. \tag{2}$$

Here, we assume that $W, T \in \mathbb{R}^{n\times n}$ are symmetric positive semidefinite matrices with at least one of them being positive definite and $i = \sqrt{-1}$. The block two-by-two linear system (1) can also be considered a special case of the generalized saddle point problem, which arises in many engineering applications, such as PDE-constrained optimization problem [1], structural dynamics [2], molecular scattering [3] and quantum mechanics [4].

Recently, many iteration methods have been constructed and studied for solving system (1), such as C-to-R iteration methods [5-8], SOR-like iteration methods [9-12], preconditioned modified Hermitian and skew-Hermitian splitting (PMHSS)iteration method [1], and other type iteration methods [13]. Here, we consider the symmetric block triangular splitting iteration method [14] proposed by using the upper and lower (ULT) iteration method and the parameterized ULT (PULT) iteration method. As we all know, the convergence rate of an iteration method depends strongly on the spectral radius of the corresponding iteration matrix. In other words, it is very vital to decrease the spectral radius of an iteration method for accelerating its convergence rate. For this purpose, Hezari et al. proposed a preconditioned GSOR (PGSOR) iteration method in reference [10], where numerical results show that the PGSOR iteration method is superior to GSOR , MHSS and HSS iteration methods in terms of iteration steps and CPU times. Motivated by the ideas in references [10,14,15], in this paper we develop a preconditioned SBTS (PSBTS) iteration method for solving system (1). And the convergence properties, the optimal relaxation parameter and the contraction factor of the PSBTS iteration method are studied. Moreover, it is proved that the spectral radius of the PSBTS iteration matrix is smaller than that of the SBTS iteration matrix under suitable conditions.

The paper is structured as follows. In Section 2, we propose the PSBTS iteration method. In Section 3, the convergence properties of the PSBTS iteration method are analyzed. In Section 4, some numerical examples are given to illustrate the effectiveness of the proposed preconditioned iteration method. Finally, brief conclusions are draw in Section 5.

*Corresponding author
Email addresses: zhangjh_496@163.com (Jianhua Zhang), zwwang6@163.com (Zewen Wang), zhaojingecut@163.com (Jing Zhao)