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Kaïs Ammari, Boumediène Chentouf

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A note on exponential and polynomial convergence for a delayed wave equation without displacement

Kaïs Ammari

UR Analysis and Control of PDE, UR13ES64, Department of Mathematics, Faculty of Sciences of Monastir, University of Monastir, 5019 Monastir, Tunisia. Email: kais.ammari@fsm.rnu.tn

Boumediène Chentouf

Kuwait University, Faculty of Science, Department of Mathematics, Safat 13060, Kuwait. Email: chenboum@hotmail.com, boumediene.chentouf@ku.edu.kw

Abstract

This article places primary emphasis on improving the asymptotic behavior of a multi-dimensional delayed wave equation in the absence of any displacement term. In the first instance, the delay is assumed to occur in the boundary. Then, invoking Bardos-Lebeau-Rauch (BLR) geometric condition [3, 7] on the domain, the exponential convergence of solutions to their equilibrium state is proved. In turn, an internal delayed wave equation is considered in the second instance, where the three-dimensional domain possesses trapped ray and hence the (BLR) geometric condition [3, 7] does not hold. Moreover, the internal damping is localized. In such a situation, polynomial convergence results are established. These two findings improve earlier results of [1, 8, 10].

1. Introduction

Given a natural number $n \geq 2$, consider an open bounded connected set of Ω in $\mathbb{R}^n$, with a sufficiently smooth boundary $\Gamma = \partial\Omega$. We assume that (Γ_0, Γ_1) is a partition of Γ. The first system, we treat in the present paper, is the wave equation with a boundary delayed damping

$$\begin{cases} y_{tt}(x,t) - \Delta y(x,t) = 0, & \text{in } \Omega \times (0,\infty), \\ \dfrac{\partial y}{\partial \nu}(x,t) = 0, & \text{on } \Gamma_0 \times (0,\infty), \\ \dfrac{\partial y}{\partial \nu}(x,t) = -\alpha y_t(x,t) - \beta y_t(x,t-\tau), & \text{on } \Gamma_1 \times (0,\infty), \\ y(x,0) = y_0(x),\ y_t(x,0) = z_0(x), & x \in \Omega, \\ y_t(x,t) = f(x,t), & (x,t) \in \Gamma_1 \times (-\tau, 0), \end{cases} \tag{1.1}$$

where ν is the unit normal of Γ pointing towards the exterior of Ω, $\alpha > 0$, and for sake for simplicity $\beta > 0$ (one may suppose that β is a nonzero real number). Assuming in the first lieu that Γ_1 is nonempty while Γ_0 may be empty, we shall invoke the geometric condition [3, 7] in the "control zone" Γ_1 in order to show that the solutions to the above system converge exponentially to an equilibrium state.

In the second lieu, we study the asymptotic behavior of a wave equation in a three-dimensional domain with a trapped ray and hence the geometric control condition is violated. Specifically, we consider the following delayed wave equation with an internal (distributed) localized damping:

$$\begin{cases} y_{tt}(x,t) - \Delta y(x,t) + a(x) y_t(x,t) + b(x) y_t(x,t-\tau) = 0, & (x,t) \in \Omega \times (0,\infty), \\ \dfrac{\partial y}{\partial \nu}(x,t) = 0, & \text{on } \Gamma \times (0,\infty), \\ y(x,0) = y_0(x),\ y_t(x,0) = z_0(x), & x \in \Omega, \\ y_t(x,t) = g(x,t), & (x,t) \in \Omega \times (-\tau, 0), \end{cases} \tag{1.2}$$

in which a is a non-negative function in $L^\infty(\Omega)$ and depends on a non-empty proper subset ω of Ω on which $1/a \in L^\infty(\omega)$ and in particular, $\{x \in \Omega; a(x) > 0\}$ is a non-empty (or of positive measure) open set of Ω, and $b \in L^\infty(\Omega)$ is a non-negative function. Assuming, as in [8], the trapping geometry on Ω, we are able to show that the solutions converge to their equilibrium state with a polynomial decay despite the presence of the delay.