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Pulse positive periodic solutions for some classes of singular nonlinearities

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Abstract

For a given nonlinear differential equations it may occur that there is no periodic solutions. By introducing impulses at prescribed instants, periodic solutions may appear. We consider some impulsive nonlinear first order differential equations having periodic solutions and extend previous results to a larger class of nonlinearities.

Keywords: Singular differential equation, Differential equation with impulses, Periodic solutions
2010 MSC: 34B16, 34A37

1. Introduction

Some evolutions processes are subject to sudden changes. The mathematical description of these processes leads to impulsive differential equations. This type of differential equations has been attracting the attention of mathematicians and they can describe population dynamics, biological phenomena or several physical situations. Moreover, impulses can be introduced on a system to generate a particular dynamic (for example periodic motions) or to control a process. We refer the reader to [1–7] for some results and applications of the impulsive differential equations.

In this paper we consider the following impulsive boundary value problem

$$\begin{aligned} &x'(t) = f(x(t)) + e(t) \\ &\Delta x(t_k) = I_k(x(t_k)),\ k = 1, \ldots, q \\ &x(0) = x(T), \end{aligned} \tag{1}$$

with $f : (0, \infty) \longrightarrow (a, b)$ a continuous function, $a \in [-\infty, +\infty)$, $b \in (-\infty, +\infty]$, $e : \mathbb{R} \longrightarrow \mathbb{R}$ a continuous and T-periodic function, $T > 0$, $I_k : \mathbb{R} \longrightarrow \mathbb{R}$, $k = 1, \ldots, q$ are continuous functions and $t_1, \ldots, t_q$ are the instants where the impulses occur, with $0 = t_0 < t_1 < \cdots < t_q < t_{q+1} = T$. We point out that a can be $-\infty$ or a real number and b can be $+\infty$ or finite.

This problem includes the following cases, among others, for the nonlinearity:

$$f(x) = x^{\beta} - 1/x^{\alpha},\ f(x) = \log(x),\ f(x) = x \sin\left(\frac{20}{x}\right) - \frac{1}{x},\ f(x) = -x^{-\alpha} - x^{-\beta} \text{ with } \alpha, \beta > 0$$

Problem (1) includes some differential equations with singularities. We refer the reader to [8–12] for some results on differential equations with singularities.

The differential equation $x'(t) = f(x(t)) + e(t)$ could have no periodic solutions. For example, if $f(x) = -1/x$ and $e \equiv 0$, then all positive solutions will be strictly decreasing and periodicity is prevented.

In this paper we obtain a result guaranteeing the existence of positive periodic solutions of the problem (1). We use a classical result due to Gaines and Mawhin [13] and we extend previous results obtained in [14, 15] by considering

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