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Optimal shape parameter in the MQ-RBF by minimizing an energy gap functional

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Abstract

By using the multiquadric radial basis function (MQ-RBF) method for solving the mixed boundary value problem as well as the Cauchy problem of the Laplace equation in an arbitrary plane domain, an energy gap functional (EGF) is proposed to choose the shape parameters. Upon minimizing the EGF we can pick up the optimal shape parameter and hence achieve the best accuracy of numerical solution. The performance of the minimizing EGF (MEGF) is assessed by numerical tests.

Keywords: Laplace equation, Cauchy problem, Energy gap functional, Minimizing energy gap functional, Optimal shape parameter

1. Introduction

Kansa [1] has applied the multiquadric radial basis function (MQ-RBF) collocation method to solve partial differential equations (PDEs). After that there are a lot of applications and developments of the MQ-RBF as an efficient meshless method to solve engineering problems. However, the ill-conditioned behavior and then the sensitivity to the shape parameter are the main obstacles in the Kansa's MQ-RBF method.

Liu et al. [2] have proposed a multiple-scale MQ-RBF method to solve the inverse Cauchy problems of the elliptic type PDEs. Many researchers have discussed the optimal shape parameter used in the MQ-RBF [3, 4, 5, 6, 7, 8]. Up to now, there does not exist a rigorous proof that the obtained shape parameter is optimal, and it is still an important issue in the MQ-RBF to determine the optimal shape parameter. Recently, Yang et al. [9] have proposed a doubly stochastic radial basis function method for function recoveries. Instead of a constant shape parameter, they treated the RBF shape parameters as stochastic variables whose distribution were determined by a stochastic leave-one-out cross validation estimation.

It is known that in the MQ-RBF the bases become flatter and the linear system to determine the expansion coefficients is more ill-conditioned when the shape parameter gets larger, during which the accuracy is improved, but eventually, the numerical solution breaks down by continuously increasing the shape parameter over a critical value. Due to the highly ill-conditioned behavior