Solving fourth order differential equations using particular solutions of Helmholtz-type equations

Wanru Chang, C.S. Chen, Wen Li

PII: S0893-9659(18)30189-7
DOI: https://doi.org/10.1016/j.aml.2018.06.012
Reference: AML 5546

To appear in: *Applied Mathematics Letters*

Received date : 8 April 2018
Revised date : 11 June 2018
Accepted date : 11 June 2018

Solving fourth order differential equations using particular solutions of Helmholtz-type equations

Wanru Chang [*], C.S. Chen [†], Wen Li[‡] [§]

Abstract

The availability of the closed-form particular solution for a given differential equation based on a chosen basis function is crucial for solving partial differential equations using the method of particular solutions. In general, the derivation of such a closed-form particular solution is by no means trivial, particularly for higher order partial differential equations. In this paper we give a simple algebraic procedure to avoid the direct derivation of the closed-form particular solutions for fourth order partial differential equations. One numerical example is given to demonstrate the effectiveness of our proposed approach.

Keywords: Particular solutions, polynomial basis functions, method of particular solutions, Helmholtz equation

1 Introduction

The method of particular solutions (MPS) was initially established as a radial basis functions (RBFs) collocation method [1]. To effectively implement the MPS, a closed-form particular solution for the given differential operator is crucial. Despite its attractive features, the MPS has encountered a major challenge in deriving the closed-form particular solution using RBFs. The closed-form particular solutions are only available for symmetric differential operators such as Laplacian, Helmholtz-type operators, etc. [5, 9, 11]. For higher order partial differential operators, the complexity of such tasks is even more pronounced. In [11], the whole article is devoted to the derivation of the closed-form particular solution for the differential operator $\Delta^2 \pm \lambda^2$ using polyharmonic splines basis functions in 2D and 3D. The resultant closed-form particular solution is quite elegant but requires a great deal of effort to obtain such closed-form particular solution.

In recent years, instead of RBFs, the MPS has extended to polynomial basis functions [2] where the closed-form particular solutions for all linear second order differential operators have become available. In the solution process, the particular solutions need to be generated using symbolic differential toolbox and then pre-stored in a table for later use. For high order of partial differential operators, the generation of the closed-form particular solution can be lengthy. In

[*]College of Mathematics, Taiyuan University of Technology, China
[†]Department of Mathematics, University of Southern Mississippi, U.S.A.
[‡]Department of Mathematics, Clarkson University, Potsdam, NY 13699-5815 and College of Big Data Science, Taiyuan University of Technology, Taiyan, China.
[§]Corresponding author: C.S. Chen, E-mail: cschen.math@gmail.com