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Structural optimization of uncertain dynamical systems considering mixed-design variables

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ABSTRACT

In this paper attention is directed to the reliability-based optimization of uncertain structural systems under stochastic excitation involving discrete–continuous sizing type of design variables. The reliability-based optimization problem is formulated as the minimization of an objective function subject to multiple reliability constraints. The probability that design conditions are satisfied within a given time interval is used as a measure of system reliability. The problem is solved by a sequential approximate optimization strategy cast into the framework of conservative convex and separable approximations. To this end, the objective function and the reliability constraints are approximated by using a hybrid form of linear, reciprocal and quadratic approximations. The approximations are combined with an effective sensitivity analysis of the reliability constraints in order to generate explicit expressions of the constraints in terms of the design variables. The explicit approximate sub-optimization problems are solved by an appropriate discrete optimization technique. The optimization scheme exhibits monotonic convergence properties. Two numerical examples showing the effectiveness of the approach reported herein are presented.

1. Introduction

For many structural optimization problems the design variables must be selected from a list of discrete values. For example, cross-sectional areas of truss members have to be chosen in general from a list of commercially available member sizes. In fact, design variables must be considered as discrete in a large number of practical design situations. Deterministic optimization procedures involving discrete design variables have been extensively studied in the literature [1–3]. Traditional methods such as branch and bound techniques, combinatorial methods, and evolution-based optimization techniques attack the discrete variable design optimization problem directly in the primal variable space [4–6]. These methods are quite general but are associated with a large number of function calls (evaluation of objective and constraint functions). Schemes for discrete structural optimization considering uncertainties have not been addressed as frequently as their deterministic counterpart [7–10]. The optimal design of uncertain structural systems under stochastic loading such as seismic excitations, water wave excitations, wind excitations, traffic loadings, etc., has been usually carried out by considering continuous design variables [11,12]. One of the difficulties in these types of problems is the high computational cost involved in the reliability analyses required during the optimization process. This is due to the fact that the reliability estimation of stochastic dynamical systems involves the estimation of failure probabilities in high-dimensional uncertain parameter spaces. In this work attention is directed to discrete–continuous robust reliability-based optimization of uncertain structural systems under stochastic excitation. Specifically, the objective of this study is to propose a general framework for performing reliability-based optimization considering discrete–continuous sizing type of design variables in an effective manner. In this context the proposed approach represents a generalization of the contribution presented in [13]. Novel aspects of this work include the incorporation of uncertain system parameters in the model, the use of an effective sensitivity analysis of reliability measures in the context of a discrete–continuous optimization scheme, and the implementation of a globally convergent optimization algorithm for solving a class of reliability-based design optimization problems.

The reliability-based optimization problem is formulated as the minimization of an objective function subject to multiple reliability constraints. The probability that design conditions are satisfied within a given time interval is used as the measure of system reliability. All uncertainties involved in the problem (system parameters and loading) are considered explicitly during the design process. Thus, final designs are robust in the sense that the optimization scheme accounts for the uncertainty in the system parameters as well as the uncertainty in the excitation. The basic mathematical programming statement of the structural optimization problem is converted into a sequence of explicit approximate primal problems. For this purpose, the objective function and the reliability constraints are approximated by using a hybrid form of linear, reciprocal and quadratic approximations.

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An approximation strategy based on an incomplete quadratic conservative approximation is considered in the present formulation [14,15]. An adaptive Markov Chain Monte Carlo procedure is used for the purpose of estimating the failure probabilities. The approach, called subset simulation, is robust to dimension size and efficient in computing small failure probabilities [16]. The information generated by subset simulation is also used to estimate the sensitivity of the reliability constraints with respect to the design variables. The above information is combined with an approximation strategy to generate explicit expressions of the objective and reliability constraints in terms of the design variables. The explicit approximate primal problems are solved either by standard methods that treat the problem directly in the primal variable space [4–6,3] or by dual methods [17,1,13]. The proposed optimization scheme exhibits monotonic convergence, that is, starting from an initial feasible design the scheme generates a sequence of steadily improved feasible designs. It ensures that the optimal solution of each approximate sub-optimization problem is a feasible solution of the original problem, with a lower objective value than the previous cycle.

First, the optimization problem, by considering the discrete–continuous sizing type of design variables is presented. The solution strategy of the problem in the framework of conservative convex and separable approximations is then discussed. Next, several implementation issues including reliability and sensitivity estimation are addressed. Finally, two numerical examples are presented.

2. Problem formulation

2.1. Optimization problem

Consider a structural optimization problem defined as the identification of a vector $\{x\}$ of design variables to minimize an objective function, that is

$$\text{Minimize } f(\{x\}) \tag{1}$$

subject to the design constraints

$$h_j(\{x\}) \leq 0, \quad j = 1, \ldots, n_c \tag{2}$$

with side constraints

$$x_i^l \leq x_i \leq x_i^u \quad i \in I_C \tag{3}$$

and

$$x_i \in X_i = \{\bar{x}_i^l, l = 1, \ldots, n_i\}, \quad i \in I_D \tag{4}$$

where I_C denotes the set of indices for continuous design variables while I_D denotes the set of indices for discrete design variables. The x_i^l and x_i^u denote the lower and upper limits for the design variables that are continuous i.e. $i \in I_C$, and Eq. (4) represents the side constraints for the design variables that are discrete i.e. $i \in I_D$. The set X_i represents the available discrete values for the design variable $x_i, i \in I_D$, listed in ascending order. It is assumed that the available values are distinct and they correspond to quantities such as cross-sectional areas, moments of inertia, etc. The particular quantity to be used depends on the problem at hand. The objective function can be defined in terms of initial costs, repair and replacement costs, downtime costs, etc. On the other hand, the design constraints can be given in terms of reliability constraints and/or deterministic design requirements. Therefore the above formulation is quite general in the sense that different reliability-based optimization formulations can be considered.

2.2. Reliability constraints

In the context of reliability-based optimization of structural systems under stochastic excitation the design constraints can be written as

$$h_j(\{x\}) = P_{F_j}(\{x\}) - P_{F_j}^* \leq 0, \quad j = 1, \ldots, n_c \tag{5}$$

where $P_{F_j}(\{x\})$ is the probability of the failure event F_j evaluated at the design $\{x\}$, and $P_{F_j}^*$ is the corresponding target failure probability. The failure probability function $P_{F_j}(\{x\})$ evaluated at the design $\{x\}$ can be expressed in terms of the multidimensional probability integral

$$P_{F_j}(\{x\}) = \int_{\Omega_{F_j}(\{x\})} q(\{\theta\})p(\{z\})\mathrm{d}\{z\}\mathrm{d}\{\theta\}, \quad j = 1, \ldots, n_c \tag{6}$$

where $\Omega_{F_j}(\{x\})$ is the failure domain corresponding to the failure event F_j evaluated at the design $\{x\}$. The failure domain can be defined in terms of a performance function κ_j as

$$\Omega_{F_j}(\{x\}) = \{\{\theta\}, \{z\} \mid \kappa_j(\{x\}, \{\theta\}, \{z\}) \leq 0\}. \tag{7}$$

where the vectors $\{\theta\}, \theta_i, i = 1, \ldots, n_U$, and $\{z\}, z_i, i = 1, \ldots, n_T$ represent the vector of uncertain structural parameters and the random variables that specify the stochastic excitation, respectively. The uncertain structural parameters $\{\theta\}$ are modeled using a prescribed probability density function $q(\{\theta\})$ while the random variables $\{z\}$ are characterized by a probability density function $p(\{z\})$. These functions indicate the relative plausibility of the possible values of the uncertain parameters $\{\theta\} \in \Omega_{\{\theta\}} \subset R^{n_U}$ and random variables $\{z\} \in \Omega_{\{z\}} \subset R^{n_T}$, respectively. The failure probability functions $P_{F_j}(\{x\}), j = 1, \ldots, n_c$ account for the uncertainty in the system parameters as well as the uncertainties in the excitation. As previously pointed out additional constraints related to deterministic design requirements can also be considered in the formulation.

2.3. First excursion probability

For systems under stochastic excitation the probability that design conditions are satisfied within a particular reference period (first excursion probability) provides a useful reliability measure. The failure events $F_j, j = 1, \ldots, n_c$ are defined as

$$F_j(\{x\}, \{\theta\}, \{z\}) = \max_{i=1,\ldots,n_j} \max_{t\in[0,T]} |s_j^i(t, \{x\}, \{\theta\}, \{z\})| \geq s_j^{i*} \tag{8}$$

where $[0, T]$ is the time interval, $s_j^i(t, \{x\}, \{\theta\}, \{z\}), j = 1, \ldots, n_c$, $i = 1, \ldots, n_j$ are the response functions associated with the failure criterion F_j, and s_j^{i*} is the corresponding critical threshold level. The performance functions for this case are given by

$$\kappa_j(\{x\}, \{\theta\}, \{z\}) = s_j^{i*} - \max_{i=1,\ldots,n_j} \max_{t\in[0,T]} |s_j^i(t, \{x\}, \{\theta\}, \{z\})|, \quad j = 1, \ldots, n_c \tag{9}$$

where the response functions $s_j^i(t, \{x\}, \{\theta\}, \{z\}), j = 1, \ldots, n_c, i = 1, \ldots, n_j$ are obtained from the solution of the equation of motion that characterizes the structural model.

3. Sequential approximate optimization

The solution of the structural optimization problem given by Eqs. (1)–(4) is obtained by transforming it into a sequence of sub-optimization problems having a simple explicit algebraic structure. Thus, the strategy is to construct successive approximate analytical sub-problems. To this end, the objective and the constraint functions are represented by using approximate functions dependent on the design variables. For the purpose of constructing the approximations all design variables are assumed to be continuous.

3.1. First-order approximation

Let $p(\{x\})$ be a generic function involved in the optimization problem, i.e. the objective or constraint functions, and $\{x^0\}$ a point in the feasible design space. The function $p(\{x\})$ is approximated